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Technical Report

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RADIOISOTOPES IN A MULTIPLE-ISOTOPE
SOURCE**

**The Imprecision of the Estimates Is Partitioned
Into Poisson, Sampling, and Mechanical Variations**

November 1967

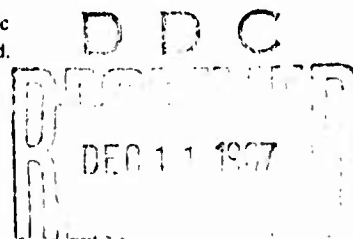
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ESTIMATING STRENGTHS OF INDIVIDUAL RADIOISOTOPES IN A MULTIPLE-ISOTOPE SOURCE

The Imprecision of the Estimates Is Partitioned Into Poisson, Sampling, and
Mechanical Variations

Technical Report R-551

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by

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ABSTRACT

In work related to radiation shielding, the use of radioisotope techniques, and activation analysis, an experimenter must often analyze counting data where counts are caused by the natural background and by the decay of more than one radioisotope. In this report a procedure is developed for estimating the strength of each isotope at different times from several decaying radioactive samples of a single multiple-isotope source. In addition, the procedure provides a method for placing confidence limits on the strengths and a method for partitioning the imprecision of estimating the strengths into three principal causes: Poisson variation, sampling error, and residual error (called mechanical error).

An operational FORTRAN II-D computer program, SAND, implements the procedure. The procedure and program were tested by using fictitious data with known properties as inputs. The results of the simulation were in reasonable agreement with the theoretical values.

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The Laboratory invites comment on this report, particularly on the
results obtained by those who have applied the information.

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INTRODUCTION

In various nuclear physics laboratories there exist dilute solutions of mixtures of a few radioactive isotopes for which an estimate is desired of the expected rate of radioactive decay* in counts per minute per given aliquot quantity of the solution at some given historic instant of time.** Usually it is desired to break this total estimate into the parts associated with each isotope.

To accomplish this, it is customary to take several aliquot samples and observe counts from each of these during measured short-time periods at several different ages (lengths of time from the desired historic instant t'_0 *** to the beginning of the measured period). Because many materials in nature, and particularly in a nuclear physics laboratory, are experiencing some radioactivity, the observing device (Geiger counter) will count slowly, even with no sample, thus, it is necessary to record counts of this background during relatively long, measured time periods near the time when the counter is to be used for sample counting. The purpose of this background count is to correct the observed counts to obtain apparent sample counts.

Given all the foregoing data, the following questions arise:

1. At the desired historic instant, designated t'_0 , what is the estimate of decay rate?
2. At t'_0 what is the breakdown of the total decay rate into the various isotope sources?

The laboratory data used to make the foregoing estimates will be subject to the following three sources of imprecision:

a. Sampling error; that is, inability to place exactly the same amount of material in exactly the same position on the watch glass each time in each finally prepared sample ready for counting and recounting.

* The terms "rate of decay" and "strength at some instant of time" are used interchangeably throughout this report.

** U.S. Naval Civil Engineering Laboratory. Technical Note N-559: Interim calibration and procedures for GM detector, by L. B. Gardner and B. Burdick, Port Hueneme, Calif., Mar. 1964.

*** The reader is referred to the Glossary of Symbols on a foldout page at the back of this report.

b. Poisson distributed variation; that is, the fact of nature that a multiplicity of samples of exactly the same size and strength (same expected counts per minute) will show variation among themselves in the actual number of radioactive events in a given minute. The nature of this Poisson fluctuation is known.

c. All other uncorrected sources of imprecision lumped under the arbitrary heading "mechanical variation." These sources may include slight positioning variation, extraneous counting errors other than from background, temperature variation of mechanical setup, slight gain or loss of material in "dried" sample between successive observations, and inexactness of dead-time, τ , correction used.

Because of a, b, and c the estimates 1 and 2 will not be exact. This raises the further questions:

3. How imprecise is the estimate of total decay rate at t'_0 ?

4. How imprecise is the estimate of each isotope decay rate at t'_0 ?

An incidental result of attempting to supply an answer to 3 will be an approximate answer to

5. How much of the imprecision in estimating decay rate at t'_0 is due to: (a) sampling error, (b) Poisson error, and (c) mechanical error?

It is the purpose of this report to develop a computational procedure for approximating answers to 1, 2, 3, 4, and 5. In short, these five questions constitute the problem to be solved in this report.

Briefly, the computational procedure will consist mainly of a sequence of parallel-weighted linear-least-squares curve fits. On each iteration each datum is weighted inversely proportional to its current estimated variance. The repetitions are terminated when the last two current estimates of the mechanical error standard deviation differ by less than some arbitrarily small value. The sampling error standard deviation is estimated by an additional iterative procedure using the foregoing results.

It is assumed that the number of isotopes in the solution and their decay constants are known; that is, the procedure does not estimate the number of isotopes in the solution, nor does it estimate the decay constants since better methods exist for this purpose.

DESCRIPTION OF MATHEMATICAL CONCEPTS

General

Several aliquot samples (total of I samples) are taken from a liquid solution of radioactive material, placed on I watch glasses, and allowed to evaporate. Then each sample is observed several times by means of a detector (J_i times for the i^{th} sample, $i = 1, 2, \dots, I$) in a fixed counting system with fixed geometry. The number of counts recorded during a period of duration T_{ij} minutes will be designated C_{ij} for the i^{th} sample and its j^{th} observation ($i = 1, 2, \dots, I; j = 1, 2, \dots, J_i$). The age of the sample in minutes, reckoned from t'_0 , an arbitrary historic instant of interest when $t = 0$, to the beginning of the period of duration T_{ij} , will be designated t_{ij} .

The assumed known number of radioactive isotopes in the source solution is L . The known decay constant in reciprocal minutes from the ℓ^{th} isotope is designated λ_ℓ ($\ell = 1, 2, \dots, L$).

Because the detector records counts from background radioactivity in addition to those strictly from the sample under observation, an effort is made to correct for this by observing background with the same instrument at a time near t_{ij} . The length of this observation and the number of counts observed are, respectively, T_{ij}^* and C_{ij}^* . In practice "near" may mean merely in the same working day. Thus, a single background reading may be used to correct several observations and give rise to T_{ij}^* and C_{ij}^* combinations which are numerically identical for various i and j combinations.

In the sampling process an effort is made to place the same amount of material on each watch glass in the same relative position. There is a slight variation in this process, and the effective quantity will be assumed to be a normally distributed random variable with mean Q and standard deviation $Q\sigma_s$, so that σ_s may be construed as a proportional standard deviation of error. For example, $\sigma_s = 0.01$ would indicate a sampling error standard deviation of 1% of the expected effective sample size.

A period of time elapses between successive observations of the same sample, a period in which laboratory conditions change. An effort is made to make all observations with exactly the same mechanical and electrical arrangement. Nevertheless, there may be a slight variation in this process which affects the actual C_{ij} recorded. If one could imagine a fictitious sample (sample number h) which would experience exactly the same number of radioactive events every minute, regardless of age, then the successive observations $C_{h1}, C_{h2}, C_{h3}, \dots$ with fixed T_{hj} and no background noise will be assumed to be items of a normally distributed random variable with mean M and standard deviation $M\sigma_m$. Now σ_m may be construed as a proportional

error due to all causes (called mechanical) other than sampling error and errors that result from the Poisson phenomenon (assumed independent of mechanical errors).

If one now imagines an infinite number of samples of the same solution of radioactive material and an infinite number of observations of each, all between t'_0 and $t'_0 + T$, for fixed T , then the background-corrected counts per minute would constitute a two-dimensional array of data entries, designated here as η_{ij} , infinite in length and infinite in width. The row (sample) mean of such a fictitious array of data will be designated $\mu_{.i}(0, T)$ counts per minute. The portion of this due to the l^{th} isotope will be designated $\mu_{l.i}(0, T)$, where

$$\mu_{.i}(0, T) = \sum_{l=1}^L \mu_{l.i}(0, T)$$

If the average of the infinity of the $\mu_{.i}(0, T)$'s is designated as $\mu_{..}(0, T)$, then question 1 of the Introduction asks for $\tilde{\mu}_{..}^0$, an estimate of $\mu_{..}^0$, where zero bias error is assumed and

$$\mu_{..}^0 = \lim_{T \rightarrow 0} \mu_{..}(0, T)$$

Similarly, question 2 asks for the L $\tilde{\mu}_{l.}^0$'s, which are estimates of the $\mu_{l.}^0$'s, where

$$\mu_{l.}^0 = \lim_{T \rightarrow 0} \mu_{l.}(0, T)$$

The mean of the i^{th} sample infinite row "at $t = 0$ " was designated as $\mu_{.i}(0, T)$, where it is supposed that

$$\mu_{.i}(0, T) = \lim_{J_i \rightarrow \infty} \frac{1}{J_i} \sum_{j=1}^{J_i} \eta_{ij}$$

and that

$$\mu_{..}(0, T) = \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i=1}^I \mu_{.i}(0, T)$$

By an observation "at $t = 0$ " is meant an observation of small duration containing $t = 0$ such that the expected counting rate at $t = 0$ is equal to the average expected counting rate during the time interval T . Recall that this is merely a fiction inserted to clarify concepts.

Briefly, the problem is to obtain estimates of $\mu_{1L}^0, \mu_1^0, \dots, \mu_L^0, \sigma_s$, and σ_m ; to compute confidence intervals expressing the imprecision in knowledge of the values of $\mu_1^0, \mu_2^0, \dots, \mu_L^0$, and μ_{1L}^0 ; and finally to separate the imprecision in estimating μ_{1L}^0 into the three sources: (a) sampling error, (b) Poisson variation, and (c) mechanical variation. To perform these estimates, it will be necessary to start the computation with $\tilde{\sigma}_{m0}, \lambda_{\ell}, t_{ij}, T_{ij}, C_{ij}, t_{ij}^*, T_{ij}^*$, and C_{ij}^* , where $\ell = 1, 2, \dots, L; j = 1, 2, \dots, J_{\ell}$; and $i = 1, 2, \dots, I$. It should be carefully noted that this whole report is concerned solely with imprecision and not laboratory mean bias over the testing period. Any such bias in μ_{1L}^0 will be carried over into $\tilde{\mu}_{1L}^0$, and as such will remain unknown in this computation.

To reach a "solution" to the problem, least-squares estimates of $\mu_{1L}^0, \dots, \mu_L^0$ for each sample will be found by using weighted linear regression. For a minimum-variance estimate, the weight used for each datum should be inversely proportional to the variance in knowledge (ignorance) of the "true" (biased by the particular sample bias) value represented by the datum. Given a particular sample, this variance for each datum is the sum of those caused by Poisson phenomenon and mechanical variation. (The last two statements taken together contain a slight inexactness stemming from a small interdependency between those elements of data which were computed using a common background reading.)

Ideally the weight of a datum should be based upon the true variance for that datum; however, only an estimate of the variance is available. The curve-fitting method depends upon the weights, which in turn depend upon the fitted parameters. Thus, it is necessary to iterate on weights and fitted parameters simultaneously for all samples to convergence of the estimated mechanical error. This leads to a stable $\tilde{\sigma}_m$ and I stable-weighted least-squares curve fits.

Preliminary Theoretical Discussion

In the foregoing, the word "expected" has been used in its technical sense and as such does not mean "anticipated." For example, consider an infinitely large quantity of radioactive solution and the division of a large portion of this with no sampling or mechanical error into a large number I of exactly equal-sized subquantities. Now imagine I background-free detector setups with absolutely no variation between the setups. At the time t_0 , initiate simultaneously the I countings (one subquantity for each counter) and terminate them all simultaneously T minutes later. This gives I integer counts C_i ($i = 1, 2, \dots, I$). Let

$$\bar{C} = \frac{1}{I} \sum_{i=1}^I C_i$$

If I is allowed to increase indefinitely, \bar{C} will approach a stable value known as the expected value of C . Restated

$$E(C) = \lim_{I \rightarrow \infty} \bar{C}$$

Thus, the expected value of a random variable is merely the average of the numerical outcomes of an infinite number of trials. The infinite number of outcomes is the population associated with the random variable. A few outcomes are a sample of the population. In general the individual C_i 's will show considerable variation among themselves. This variation of the population of counts has been called the Poisson phenomenon. Indeed, the a priori probability that one particular counter will count exactly h radioactive events during the period t'_0 to $t'_0 + T$ is given by the Poisson formula

$$P(C_i = h) = \frac{e^{-E(C)} [E(C)]^h}{h!} \quad \text{for } h = 0, 1, 2, \dots$$

Now if the variance of C is defined to be

$$\sigma_C^2 = E\{[C - E(C)]^2\}$$

it will be found that

$$\sigma_C^2 = E(C)$$

or equivalently that the standard deviation of counts

$$\sigma_C = +\sqrt{E(C)}$$

which is a very interesting property of a Poisson distributed random variable such as C . Restated

$$\sigma_C^2 = \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i=1}^I [C_i - E(C)]^2$$

Because counts per minute for no detector deadtime, $\tau = 0$, is $N = C/T$, T merely plays the role of a scale factor with the following consequences:

$$E(N) = E\left(\frac{C}{T}\right) = \frac{1}{T} E(C)$$

$$\sigma_N^2 = E\left\{\left[\frac{C}{T} - E\left(\frac{C}{T}\right)\right]^2\right\} = \frac{1}{T^2} E\{[C - E(C)]^2\}$$

$$= \frac{\sigma_C^2}{T^2} = \frac{E(C)}{T^2} = \frac{E(N)}{T}$$

From above $\sigma_N = \frac{1}{T} \sigma_C = \frac{1}{T} \sqrt{E(C)} = \sqrt{\frac{E(N)}{T}}$

Of course in a real-world laboratory it is not possible to perform the idealized experiment of this section because there may be only one detector. In such a case several samples may be observed at different ages, or each sample may be reobserved at different ages. For a fixed period length of T minutes the expected counts will decrease or decay with age. Fortunately, the physical law governing this decay of expectation is known. If at time t_0 the instantaneous expected background-free counts per minute are

$$\mu_{..}^0 = \sum_{\ell=1}^L \mu_{\ell}^0$$

then at any future instant t

$$\mu'_{..}(t) = \sum_{\ell=1}^L \mu_{\ell}^0 e^{-\lambda_{\ell} t}$$

and over a period from t to $t + T$ the expected counts, assuming $\tau = 0$ (perfect detector), are

$$E(C) = \left[\int_t^{t+T} \mu'_{..}(t) dt \right] + E(C^*)$$

where C^* is the actual counts from background during T .

Equivalently, the expected counts per minute solely from sample are

$$E(\eta) = \frac{1}{T} \int_0^T \mu'_\cdot(t) dt = \sum_{\ell=1}^L \mu_{\ell}^0 e^{-\lambda_{\ell} T} \left(\frac{1 - e^{-\lambda_{\ell} T}}{\lambda_{\ell} T} \right)$$

Letting
$$X(t, T; \lambda_{\ell}) = e^{-\lambda_{\ell} t} \left(\frac{1 - e^{-\lambda_{\ell} T}}{\lambda_{\ell} T} \right)$$

then
$$E(\eta) = \sum_{\ell=1}^L \mu_{\ell}^0 X(t, T; \lambda_{\ell})$$

For the j^{th} observation ($j = 1, 2, \dots, J_i$) of the i^{th} sample

$$E(\eta_{ij}) = \sum_{\ell=1}^L \mu_{\ell i}^0 X(t_{ij}, T_{ij}; \lambda_{\ell})$$

and
$$\eta_{ij} = \sum_{\ell=1}^L \mu_{\ell i}^0 X(t_{ij}, T_{ij}; \lambda_{\ell}) + \epsilon_{ij}$$

where ϵ_{ij} is a random error consisting of the algebraic sum of the random Poisson sample error (reckoned from expected counts per minute), the random Poisson background error (reckoned from expected background counts per minute), and the random mechanical error.

Thus the estimates of

$$E(\eta_{ij}) = \sum_{\ell=1}^L \mu_{\ell i}^0 X_{\ell ij}$$

can be obtained from a weighted least-squares fit of $L + 1$ columns of data (each J_i long). The column headings are $\eta_i, X_{1i}, X_{2i}, \dots, X_{Li}$ and the corresponding entries are $\eta_{ij}, X(t_{ij}, T_{ij}; \lambda_{\ell}) \equiv X_{\ell ij}$, where $j = 1, 2, \dots, J_i$ and $\ell = 1, 2, \dots, L$. Each of the J_i lines of data will be weighted at each iteration inversely proportional to the last iterated estimate of $E(\epsilon_{ij}^2)$. After studying the next section, the careful reader will note that the foregoing sentence is very slightly inexact. Because a single background reading may be used for more than one combination of i and j , the next section deliberately modifies the method of weighting so as to reckon each background count once in the whole data-weighting computation system.

Clearly, one will never be able to perform enough countings to display the whole population of outcomes, but must be content to perform several tests, giving merely a sample of several numerical valued outcomes from the population. The average of the sample of outcomes is called a sample mean and as such constitutes an estimate of the population mean. For example

$$\bar{C} = E(\bar{C})$$

In general this is the best available estimate without further testing. Nonetheless, it may be a poor estimate. It is customary to quantify the poorness or goodness of the estimate by computing a confidence interval at some expressed level of confidence for the quantity estimated. A 0.95 interval for $E(\bar{C})$ is a range of possible values with specified lower and upper end points where the interval has the following interpretation: "The true value of $E(\bar{C})$ is not known; however we are 95% sure that whatever its value may be it does lie in this range." Thus, as the precision of the estimate increases, the confidence interval decreases in length. In this report answers to questions 3 and 4 of the Introduction are expressed as 0.95 confidence intervals.

PROBLEM SOLUTION

The broad plan for solution will consist of four main steps:

1. All I of the $\tilde{\mu}_i^0$ are computed. Each of these computations uses the J_i observations associated with the particular sample; $\tilde{\sigma}_{mo}$, the historic estimate of proportional mechanical error standard deviation with f_0 associated degrees of freedom; and a least-square fit to the J_i weighted points to arrive at the estimates of

$$\mu_{.i}^0 = \sum_{\ell=1}^L \mu_{\ell i}^0$$

where it is assumed that

$$\mu_{\ell i}^0 = \lim_{T \rightarrow 0} \mu_{\ell i}(0, T) \quad \text{and} \quad \mu_{.i}^0 = \lim_{T \rightarrow 0} \mu_{.i}(0, T)$$

Estimates of σ_m based upon the sum of historic and current degrees of freedom are used in the computation for weighting. To obtain $\tilde{\sigma}_m$ all I samples are used. Thus, all I least-square fits are performed in parallel using the results of the k^{th}

iteration to obtain all the $\tilde{\mu}_{\ell ik}^o$ (l times L of these) and $\tilde{\sigma}_{mk}$ to be used in the (k + 1)th iteration. The magnitude of σ_s , the proportional sampling error standard deviation, in no way influences these computations because each fit is for a fixed single sample.

2. The l final $\tilde{\mu}_{\ell i}^o$'s; $\tilde{\sigma}_{so}$, the historic estimate of σ_s with d associated degrees of freedom; and $S_{\tilde{\mu}_{\ell i}^o}^2$, the estimated variance of $\mu_{\ell i}^o$, are used to compute a new $\tilde{\sigma}_s$ by iteration.

3. The statistics from the foregoing final iterations are used for computing confidence intervals, and for partitioning total uncertainty in $\tilde{\mu}_{\ell i}^o$ into its three basic components.

4. The final $\tilde{\sigma}_m$ and $\tilde{\sigma}_s$, based on both current and past data, together with their respective associated degrees of freedom, are retained as inputs to future data analysis.

Estimation of $\mu_{\ell i}^o$ and σ_m

The linear mathematical model is

$$\mu_{\ell i}(t_{ij}, T_{ij}) = \sum_{\ell=1}^L \mu_{\ell i}^o X_{\ell ij}$$

where the transformed independent variables are

$$X_{\ell ij} = e^{-\lambda_{\ell} t_{ij}} \left(\frac{1 - e^{-\lambda_{\ell} T_{ij}}}{\lambda_{\ell} T_{ij}} \right)$$

with the corresponding linear statistical model

$$\eta_{ij} = \sum_{\ell=1}^L \mu_{\ell i}^o X_{\ell ij} + \epsilon_{ij}$$

where the transformed dependent variables are

$$\eta_{ij} = \frac{C_{ij}}{T_{ij} \left(1 - r \frac{C_{ij}}{T_{ij}} \right)} - \frac{C_{ij}^*}{T_{ij}^*}$$

and ϵ_{ij} is the jth deviation for the ith sample.

The strength of each isotope for the i^{th} sample is estimated by the weighted least-squares method, where each weight is inversely proportional to the variance of the datum. Since the variances are unknown, they must be estimated from past and current data. This leads to a nonlinear problem which is solved iteratively by starting with some initial estimates for the weights.

Using estimated weights, for each i at each iteration, minimize

$$\sum_{j=1}^{J_i} w_{ijk} \left[\eta_{ij} - \tilde{\mu}_{.ik}(t_{ij}, T_{ij}) \right]^2 = \sum_{j=1}^{J_i} w_{ijk} \left(\eta_{ij} - \sum_{\ell=1}^L \tilde{\mu}_{\ell ik}^o x_{\ell ij} \right)^2$$

with respect to $\tilde{\mu}_{\ell ik}^o$ to obtain estimates $\tilde{\mu}_{\ell ik}^o$ by solving the resulting L simultaneous linear equations.

The estimation of weights at each iteration requires the partitioning of the estimated variance into its components. As a consequence, a new estimate of the sampling error σ_s is obtained.

An estimate of the total variance due to both mechanical and Poisson sources for j^{th} observation of the i^{th} sample is the usual

$$\frac{S_{ik}^2}{w_{ijk}} = \frac{1}{J_i - L} \sum_{j=1}^{J_i} w_{ijk} \left(\eta_{ij} - \sum_{\ell=1}^L \tilde{\mu}_{\ell ik}^o x_{\ell ij} \right)^2$$

There are two causes for each deviate

$$\eta_{ij} - \tilde{\mu}_{.ik}(t_{ij}, T_{ij})$$

namely, the error variance stemming from Poisson phenomena estimated to be

$$\frac{N_{ij}^* + \tilde{\mu}_{.ik}(t_{ij}, T_{ij})}{T_{ij}} + \frac{N_{ij}^*}{T_{ij}^*}$$

plus the error variance stemming from mechanical causes estimated to be

$$\left[\tilde{\sigma}_{mk} \tilde{\mu}_{.ik}(t_{ij}, T_{ij}) \right]^2$$

A point by point partitioning of each deviate into amounts ascribable to mechanical and to Poisson sources is performed to obtain a point estimate of the current sampling error σ_m . These point estimates are averaged over

all current observations to get $\tilde{\sigma}_{m'k}^2$, which, together with the historic estimate $\tilde{\sigma}_{mo}^2$, are averaged to provide an updated historic estimate for the next iteration. The foregoing procedures, as well as those for estimating the weights and source strengths, are summarized in the following simultaneous equations:

For $k = 0, 1, 2, \dots, \text{last}$

$$\left. \begin{aligned} w_{ijk+1} &= \frac{K_{i1}}{\frac{N_{ij}}{T_{ij}} + \frac{N_{ij}^*}{T_{ij}^*} + (\tilde{\sigma}_{mo} \eta_{ij})^2} & \text{for } k = 0 \\ &= \frac{K_{ik+1}}{\frac{N_{ij}^* + \tilde{\mu}_{..k}(t_{ij}, T_{ij})}{T_{ij}} + \frac{N_{ij}^*}{T_{ij}^*} + [\tilde{\sigma}_{mk} \tilde{\mu}_{..k}(t_{ij}, T_{ij})]^2} & \text{for } k > 0 \end{aligned} \right\} \quad (1)$$

where K_{ik} is consistent with

$$\sum_{i=1}^{J_i} w_{ijk+1} = 1 \quad (2)$$

In matrix notation, for $h = 1, 2, \dots, L$,

$$\begin{aligned} \left(\tilde{\mu}_{\ell ik+1}^o \right)_{L \times 1} &= \left(\sum_{j=1}^{J_i} w_{ijk+1} X_{hij} X_{\ell ij} \right)_{L \times L}^{-1} \left(\sum_{j=1}^{J_i} w_{ijk+1} X_{\ell ij} \eta_{ij} \right)_{L \times 1} \\ \tilde{\sigma}_{m'k+1}^2 &= \frac{\sum_{i=1}^I (J_i - L) \left\{ \frac{J_i}{J_i - L} \sum_{j=1}^{J_i} w_{ijk+1} \left[\frac{\eta_{ij} - \tilde{\mu}_{..k+1}(t_{ij}, T_{ij})}{\tilde{\mu}_{..k+1}(t_{ij}, T_{ij})} \right]^2 \Omega \right\}}{\sum_{i=1}^I (J_i - L)} \quad (3) \end{aligned}$$

$$\text{where } \Omega = \frac{[\tilde{\sigma}_{mk} \tilde{\mu}_{..k+1}(t_{ij}, T_{ij})]^2}{\left[\tilde{\sigma}_{mk} \tilde{\mu}_{..k+1}(t_{ij}, T_{ij}) \right]^2 + \frac{\tilde{\mu}_{..k+1}(t_{ij}, T_{ij}) + N_{ij}^*}{T_{ij}} + \frac{N_{ij}^*}{T_{ij}^* U_{ij}}}$$

$$\tilde{\sigma}_{m'k+1}^2 = \frac{f_0 \tilde{\sigma}_{m0}^2 + f_1 \tilde{\sigma}_{m'k+1}^2}{f_0 + f_1} \quad (4)$$

where $\tilde{\sigma}_{m0}$ is the historic estimate of the proportional mechanical error based on f_0 degrees of freedom, and

$$f_1 = \sum_{i=1}^I (J_i - L)$$

is the additional degrees of freedom available from the current data.

The introduction of the integers U_{ij} associated with background reading requires explanation. A single background reading may at times be used to correct more than one observation of the same sample or even of different samples. The product of the number of samples for which a background reading has been used to compute η_{ij} 's times the number of observations of the i th sample for which this particular background reading has been used is designated U_{ij} . It is introduced into the iteration merely to assure that the variance due to a single background reading gets counted only once in the averaging process over all observations for all samples.

Each deviate was divided by $\tilde{\mu}_{ik}(t_{ij}, T_{ij})$ to assure that $\tilde{\sigma}_m$ will be on a proportional mechanical error basis, as defined. The last multiplier in Equation 3 partitions each squared deviate into the fraction blamed on mechanical error. The sum of the $J_i - L$ (degrees of freedom associated with i th sample) appearing in the denominator and the multipliers $J_i - L$ in the numerator average $\tilde{\sigma}_m^2$ over all the total

$$\sum_{i=1}^I (J_i - L)$$

degrees of freedom available.

Iteration is terminated when two successive $\tilde{\sigma}_m$ differ by less than an arbitrarily small amount, δ , say 0.00005 (0.005%); that is,

$$|\tilde{\sigma}_{m'k+1} - \tilde{\sigma}_{m'k}| < \delta$$

At the end of the least-square fitting process, in addition to the final $\tilde{\sigma}_m$, there are available for each sample the final $L + 1$ estimates contained in

$$\tilde{\mu}_{\cdot i}^0 = \sum_{\ell=1}^L \tilde{\mu}_{\ell i}^0$$

These $\tilde{\mu}_{0,i}^o$ and I final inverse matrices with associated S_i^2 from the least-square fitting process are used in establishing confidence intervals and in estimating σ_s .

Estimation of $\mu_{..}^o$ and σ_s

The I samples have been used to obtain the final estimates $\tilde{\mu}_{.1}^o, \tilde{\mu}_{.2}^o, \dots, \tilde{\mu}_{.I}^o$, together with the I -associated final L by L inverse matrices and S_i^2 used for the least-squares fits.

In using these it will be assumed erroneously that the estimates $\tilde{\mu}_{.1}^o, \tilde{\mu}_{.2}^o, \dots, \tilde{\mu}_{.I}^o$ are independent. The slight dependency that may exist between two particular $\tilde{\mu}_{.i}^o$'s stems solely from the use of some of the same background readings for the computation of each of the two $\tilde{\mu}_{.i}^o$'s. This dependency is negligible in general because almost invariably $N_{ij}^* < N_{ij}$. To consider the dependency would unduly complicate the computation.

From the last iteration of the preceding section

$$S_{\tilde{\mu}_{.i}^o}^2 = A_i S_i^2$$

where A_i is the sum of all the L^2 elements of the last inverse matrix for sample i .

To obtain $\tilde{\mu}_{..}^o$ and $\tilde{\mu}_{..}^o \tilde{\sigma}_s$, weights v_1, v_2, \dots, v_I will be iterated with the I -fixed $S_{\tilde{\mu}_{.i}^o}^2$ and the I -fixed $\tilde{\mu}_{.i}^o$ as inputs to obtain a stable estimate of $\tilde{\sigma}_s$. The

iteration algorithm to accomplish this consists of the successive use of the following simultaneous equations. For $k = 1, 2, 3, \dots, \text{last}$

$$\left. \begin{aligned} v_{ik} &= \frac{\kappa_1}{(\tilde{\mu}_{.i}^o \tilde{\sigma}_{s0})^2 + S_{\tilde{\mu}_{.i}^o}^2} & \text{for } k = 1 \\ &= \frac{\kappa_k}{(\tilde{\mu}_{..,k-1}^o \tilde{\sigma}_{s,k-1})^2 + S_{\tilde{\mu}_{.i}^o}^2} & \text{for } k > 1 \end{aligned} \right\} \quad (5)$$

where κ_k is consistent with

$$\sum_{i=1}^I v_{ik} = 1$$

$$\tilde{\mu}_{..k}^o = \sum_{i=1}^I v_{ik} \tilde{\mu}_{.i}^o \quad (6)$$

$$\tilde{\sigma}_{s'k}^2 = \frac{1}{I-1} \sum_{i=1}^I v_{ik} \left(\frac{\tilde{\mu}_{.i}^o - \tilde{\mu}_{..k}^o}{\tilde{\mu}_{..k}^o} \right)^2 \frac{(\tilde{\mu}_{..k}^o \tilde{\sigma}_{sk-1})^2}{(\tilde{\mu}_{..k}^o \tilde{\sigma}_{sk-1})^2 + S_{\tilde{\mu}_{.i}^o}^2} \quad (7)$$

$$\tilde{\sigma}_{sk}^2 = \frac{(I-1) \tilde{\sigma}_{s'k}^2 + d \tilde{\sigma}_{so}^2}{d + I - 1} \quad (8)$$

where $\tilde{\sigma}_{so}$ is the historic estimate of proportional sampling error based on d degrees of freedom. The historic estimate stems from previous analyses of all pertinent counting data from a laboratory before the

$$\sum_{i=1}^I J_i$$

observations currently being analyzed.

Estimation of μ_{ℓ}^o

Inputs to the estimation of μ_{ℓ}^o .

1. v_1, v_2, \dots, v_I from the last iteration of the preceding section.
2. The $\tilde{\mu}_{\ell i}^o$ (L of these) from the final iteration of the section entitled "Estimation of $\mu_{\ell i}^o$ and σ_m ."

Each of the $\tilde{\mu}_{\ell}^o$ will be computed as a simple partitioning of the $\tilde{\mu}_{..}^o$ so that

$$\tilde{\mu}_{..}^o = \sum_{\ell=1}^L \tilde{\mu}_{\ell}^o$$

as follows:

$$\tilde{\mu}_{\ell}^o = \sum_{i=1}^I v_i \tilde{\mu}_{\ell i}^o$$

This partitioning is based not on statistical grounds, but on the physical fact that all samples of equilibrium solutions of mixtures contain the same ratio of ingredients, to a precision on the order of the reciprocal of the square root of the number of molecules involved in the ingredient sample.

Confidence Intervals for $\mu_{..}^o$ and $\mu_{Q.}^o$

Let t^* be a value computed such that

$$\int_{-t^*}^{t^*} f(t) dt = 0.95$$

where $f(t)$ is the Student's t density function for t with $I - 1$ degrees of freedom. Then the 0.95 confidence interval for $\mu_{..}^o$ extends over the interval

$$\tilde{\mu}_{..}^o \pm t^* \sqrt{\frac{\sum_{i=1}^I v_i (\tilde{\mu}_{.i}^o - \tilde{\mu}_{..}^o)^2}{I - 1}}$$

and for $\mu_{Q.}^o$ over the interval

$$\tilde{\mu}_{Q.}^o \pm t^* \sqrt{\frac{\sum_{i=1}^I v_i (\tilde{\mu}_{Q.i}^o - \tilde{\mu}_{Q.}^o)^2}{I - 1}}$$

Partitioning $\tilde{\sigma}_{\tilde{\mu}_{..}^o}^2$

The estimated variance in estimating $\tilde{\mu}_{..}^o$ is

$$\tilde{\sigma}_{\tilde{\mu}_{..}^o}^2 = \frac{1}{I - 1} \sum_{i=1}^I v_i (\tilde{\mu}_{.i}^o - \tilde{\mu}_{..}^o)^2$$

which has three components that are assumed independent. These components are errors attributed to sampling, mechanical difficulties, and Poisson phenomena, and may be expressed as

$$\tilde{\sigma}_{\tilde{\mu}_{..}^o}^2 = \left(\tilde{\sigma}_{\tilde{\mu}_{..}^o}^2 \right)_s + \left(\tilde{\sigma}_{\tilde{\mu}_{..}^o}^2 \right)_m + \left(\tilde{\sigma}_{\tilde{\mu}_{..}^o}^2 \right)_p$$

The computation of these three components will now be discussed. A crude estimate of

$$\left(\tilde{\sigma}_{\tilde{\mu}_{..}^o}^2 \right)_m + \left(\tilde{\sigma}_{\tilde{\mu}_{..}^o}^2 \right)_p$$

is

$$\sum_{i=1}^I v_i^2 S_{\tilde{\mu}_{..}^o}^2 = \sum_{i=1}^I v_i^2 A_i S_i^2$$

where A_i is the sum of all the L^2 elements in the last inverse matrix for sample i and

$$S_i^2 = \frac{1}{J_i - L} \sum_{j=1}^{J_i} w_{ij} \left[\eta_{ij} - \tilde{\mu}_{..i}(t_{ij}, T_{ij}) \right]^2$$

To partition S_i^2 into its Poisson and mechanical components and simultaneously assure that the sum of the parts equals the whole, it is necessary to partition the individual squared deviates. The point estimate for the j^{th} observation on the i^{th} sample for the variation due to the Poisson phenomenon is

$$a_{ij} = \frac{\tilde{\mu}_{..i}(t_{ij}, T_{ij}) + N_{ij}^*}{T_{ij}} + \frac{N_{ij}^*}{U_{ij} T_{ij}^*}$$

and for the variation due to mechanical causes is

$$\beta_{ij} = \left[\tilde{\sigma}_m \tilde{\mu}_{..i}(t_{ij}, T_{ij}) \right]^2$$

then

$$\left(\tilde{\sigma}_{\tilde{\mu}_{..i}^o}^2 \right)_p = \frac{A_i}{J_i - L} \sum_{j=1}^{J_i} w_{ij} \left[\eta_{ij} - \tilde{\mu}_{..i}(t_{ij}, T_{ij}) \right]^2 \frac{a_{ij}}{a_{ij} + \beta_{ij}}$$

$$\left(\tilde{\sigma}_{\tilde{\mu}_{..i}^o}^2 \right)_m = \frac{A_i}{J_i - L} \sum_{j=1}^{J_i} w_{ij} \left[\eta_{ij} - \tilde{\mu}_{..i}(t_{ij}, T_{ij}) \right]^2 \frac{\beta_{ij}}{a_{ij} + \beta_{ij}}$$

which guarantees that

$$S_{\tilde{\mu}_{..i}^o}^2 = \left(\tilde{\sigma}_{\tilde{\mu}_{..i}^o}^2 \right)_m + \left(\tilde{\sigma}_{\tilde{\mu}_{..i}^o}^2 \right)_p$$

Hence, define

$$a_{..} = \sum_{i=1}^I v_i^2 \left(\tilde{\sigma}_{\mu_{..i}}^2 \right)_p$$

$$\beta_{..} = \sum_{i=1}^I v_i^2 \left(\tilde{\sigma}_{\mu_{..i}}^2 \right)_m$$

where $a_{..}$ and $\beta_{..}$ are crude estimates of $\left(\tilde{\sigma}_{\mu_{..}}^2 \right)_p$ and $\left(\tilde{\sigma}_{\mu_{..}}^2 \right)_m$, respectively.

There is no guarantee that $a_{..} + \beta_{..}$ may not exceed $\tilde{\sigma}_{\mu_{..}}^2$, because through the vagaries of chance the I values of $\tilde{\mu}_{..i}^o$ may be unusually close together.

Let
$$\gamma_{..} = (\tilde{\mu}_{..}^o \tilde{\sigma}_s)^2 \sum_{i=1}^I v_i^2$$

Finally
$$\left(\tilde{\sigma}_{\mu_{..}}^2 \right)_p = \frac{a_{..}}{a_{..} + \beta_{..} + \gamma_{..}} \tilde{\sigma}_{\mu_{..}}^2 \quad (9)$$

$$\left(\tilde{\sigma}_{\mu_{..}}^2 \right)_m = \frac{\beta_{..}}{a_{..} + \beta_{..} + \gamma_{..}} \tilde{\sigma}_{\mu_{..}}^2 \quad (10)$$

$$\left(\tilde{\sigma}_{\mu_{..}}^2 \right)_s = \frac{\gamma_{..}}{a_{..} + \beta_{..} + \gamma_{..}} \tilde{\sigma}_{\mu_{..}}^2 \quad (11)$$

Summary

The Problem Solution section may be summarized briefly in the following manner:

1. Transform the raw data as specified in the section entitled, "Estimation of $\mu_{\bar{Q}}^o$ and σ_m ."

2. For each i compute a least-square fit of the form

$$\tilde{\mu}_{.i}(t_{ij}, T_{ij}) = \tilde{\mu}_{1i} X_{1ij} + \tilde{\mu}_{2i} X_{2ij} + \dots + \tilde{\mu}_{Li} X_{ij}$$

3. From these I results estimate σ_m^2 by using Equation 3.

4. Use Equation 4 to compute $\tilde{\sigma}_{m2}$ and then an array of new weights w_{ij2} .

5. Continue this 2 - 3 - 4 iterative sequence until

$$|\tilde{\sigma}_{m'k} - \tilde{\sigma}_{m'k-1}| < 5 \times 10^{-5}$$

6. From the last iteration obtain $\tilde{\sigma}_m$, $\tilde{\sigma}_m$, $f_o + f_1$, all $\tilde{\mu}_{\ell i}^o$, all $\tilde{\mu}_{.i}^o$, and all $S_{\tilde{\mu}_{.i}^o}^2$.

7. Using the iterative scheme given by Equations 5 through 8, obtain $\tilde{\mu}_{..}^o$, $\tilde{\sigma}_s$, $\tilde{\sigma}_s$, and $d + 1 - 1$ on convergence; that is, such that

$$|\tilde{\sigma}_{s'k} - \tilde{\sigma}_{s'k-1}| < 5 \times 10^{-5}$$

8. Using the weights, v_i 's, of the last iteration in step 7, compute all $\tilde{\mu}_{\ell}^o$ with 0.95 confidence limits on $\tilde{\mu}_{..}^o$ and each $\tilde{\mu}_{\ell}^o$.

9. Use Equations 9, 10, 11 to partition the estimated variance of $\tilde{\mu}_{..}^o$ into the components attributed to Poisson, mechanical, and sampling causes.

DISCUSSION OF MATHEMATICAL MODEL

The procedure outlined in this report estimates, simultaneously, total random-and-systematic error, its Poisson component, its sampling component, and its residual component, called mechanical error (a presumed composite from all causes other than sampling variation and Poisson phenomenon). This analytical procedure is expressed in terms of the error components at time t'_0 , that is, age zero. The error under discussion is error in estimating corrected system counts per minute for an exact intended aliquot.

In principle, this unified simultaneous approach should be superior to separate estimations of the three foregoing components of error. In passing, it should be noted clearly that this report is concerned with an analysis, not a calibration; that is, laboratory biases will never be brought to light by this procedure alone. The analysis may, however, be used for the calibration of a laboratory when laboratory data from a known standard solution mixture are available.

Because the procedure developed in this report is an initial effort, it is not the last word. An effort was made to obtain either unbiased estimates, maximum likelihood estimates, or both, so that the results could be claimed to be "best" in some sense. Unfortunately, such an effort did not seem feasible at all points of the development. The slight statistical imperfections that have crept into the work will now be discussed.

The mathematical model assumes that the standard deviation in counts per minute for the random error from all causes other than sampling and Poisson phenomenon is proportional strictly to expected corrected counts per minute regardless of age, isotope solution component strengths, or length of observing period. Perhaps this is not true in practice. All other unaccounted-for errors are encompassed by σ_m , so that $(\mu - \sigma_m)^2$ plays the role of a residual. Nevertheless, the unitless proportionality constant σ_m is very meaningful for the physicist. The statistician will note that σ_m is the coefficient of variation for "other causes," assumed, perhaps erroneously, to be constant.

On the other hand, the model $\mu \propto \sigma_s$ for standard deviation of sampling random error should be almost true in fact.

In computing the variance of a datum, expressed as counts-per-minute squared, heavy use has been made of the fact that for a Poisson density of counts, the mean and variance are equal. Statisticians generally accept this "fact," but the physicist will note immediately that the units are wrong; counts cannot equal the square of the number of counts. To make physical sense, the statisticians' equation needs a unit, u^* , with dimension reciprocal-counts so that

$$\mu = u^* \sigma^2$$

which translated says, "for the population of a fixed Poisson process, the positive square root of the numerical value of the mean expressed as a pure number is the same as the standard deviation of the population expressed as a pure number." The unit u^* has been deliberately left out of the Problem Solution section. To check on unit consistency of the equations, u^* will need to be introduced.

The Student's t density has been freely used in computing confidence intervals, even though it is realized that the uncertainty in estimating the various μ^o 's is not strictly represented by a normal distribution, since some of this uncertainty stems from nonnormal Poisson processes. It is believed that use of the Student's t does not result in very serious errors. This belief is based upon the rapid convergence usually encountered in applying the central limit theorem and upon the fact that the μ 's, J_i 's, and particularly

$\sum_{i=1}^I J_i$ are sufficiently large to nearly normalize the uncertainty.

Criticism could be directed at the following type of sequential steps which were employed freely:

$$1. \sigma_{a+b}^2 = \sigma_a^2 + \sigma_b^2$$

$$2. \tilde{\sigma}_{a+b}^2 = \tilde{\sigma}_a^2 + \tilde{\sigma}_b^2$$

In the first place, 1 is not true unless the random variables a and b are independent of each other. Where 1 was used the two random variables involved were only very nearly independent. The step 1 to 2 is not valid in general unless the quantities in 2 are unbiased.

Additional criticism might be directed at partitioning for $\tilde{\sigma}_m$ by ratios rather than differences; partitioning $\tilde{\sigma}_s$ is, of course, open to the same criticism. This method of partitioning was employed to avoid negative $\tilde{\sigma}_m^2$ and negative $\tilde{\sigma}_s^2$, since such results are both impossible and meaningless in practice and cause insurmountable difficulties in using weights based on estimated variances. The extent of possible bias introduced by this method of partitioning is unknown. An adequate treatment of estimated weights or of partitioning variances into their various components by the ratio method is not given in the literature, and investigation of these subjects would constitute a larger study.

In spite of the foregoing reservations it is believed that this combined unified approach will prove to be a significant improvement over piecemeal analyses, and a vast improvement over assuming that all variation is Poisson. Use of the combined approach will certainly enable a laboratory to keep records of its $\tilde{\sigma}_s$ and $\tilde{\sigma}_m$ estimates with the hope that these estimates will drift lower with the age of the laboratory as techniques improve. Certainly these estimates will point to the area where the most improvement is to be made. Should σ_s and σ_m regress significantly with time, their regression estimates would be the ones to use in the weighting process.

It is noted that under many sets of conditions even the best estimate has an appreciable chance of missing considerably. For the problem of this report, the following will tend to improve the estimate of expected decay rate at some historic time of interest:

1. Decrease of sampling error.
2. Sampling replication.
3. Decrease of other errors.
4. Replicating and lengthening periods of observing samples.

Most notable is item 2. Presuming reasonable efforts in areas 1 and 3 have been made, but an appreciable sampling error persists, the greatest gain per unit of effort likely can be accomplished by increasing the number of samples of the isotope solution. Simply replicating the number of observations of a given sample will not average out sampling fluctuation. When records of $\tilde{\sigma}_m$ and $\tilde{\sigma}_s$ are in hand it will be possible to use them to design a schedule for minimum-variance estimate within a fixed total cost.

SIMULATION

A FORTRAN program was written to implement the procedure given in the Problem Solution section. It is presented and discussed in the Appendix. How well does this procedure work? In order to obtain a preliminary answer to this, it is desirable to operate the program using input data from a decay phenomenon with known parameters. To accomplish this it was necessary to develop a data generator.

Data Generator

The known constants and parameters required for generating data are λ_0 , L , μ_0^0 , σ_s , σ_m , I , J_i , t_{ij} , T_{ij} , T_{ij}^* , and B_{ij} (expected background counts per minute).

The objective is to simulate I samples and create J_i observations ($i = 1, 2, \dots, I$) at different ages of the samples, assuming a constant parameter for the Poisson-distributed background, and also to create an independent set of background observations. This is accomplished in the following steps.

Step 1: I random numbers are drawn from a normal population with mean zero and standard deviation σ_s . Name these r_1, r_2, \dots, r_I . For each i compute:

$$\mu_{1i}^0 = (1 + r_i) \mu_1^0$$

$$\mu_{2i}^0 = (1 + r_i) \mu_2^0$$

$$\begin{matrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{matrix}$$

$$\mu_{Li}^0 = (1 + r_i) \mu_L^0$$

Step 2: For each of the I samples draw J_i random numbers from a normal population with mean zero and standard deviation σ_m . Name these

$q_{i1}, q_{i2}, \dots, q_{iJ_i}$. Compute $\sum_{i=1}^I J_i$ quantities

$$\xi_{ij} = (1 + q_{ij}) \left[\mu_{1i}^0 \left(\frac{1 - e^{-\lambda_1 T_{ij}}}{\lambda_1 T_{ij}} \right) e^{-\lambda_1 t_{ij}} + \dots + \mu_{Li}^0 \left(\frac{1 - e^{-\lambda_L T_{ij}}}{\lambda_L T_{ij}} \right) e^{-\lambda_L t_{ij}} \right]$$

Step 3: With each of the T_{ij} ($\xi_{ij} + B_{ij}$) as the Poisson parameter, select at random one count and name it C_{ij} . Select at random one count from each Poisson density with parameter $B_{ij} T_{ij}^*$ and name it C_{ij}^* .

Fifteen sets of data were generated. For each set the input constants and parameters were

$$\lambda_1 = 0.30/\text{min}$$

$$\lambda_2 = 0.15/\text{min}$$

$$\lambda_3 = 0.05/\text{min}$$

$$\mu_1^0 = 8,000 \text{ cpm}$$

$$\mu_2^0 = 1,500 \text{ cpm}$$

$$\mu_3^0 = 500 \text{ cpm}$$

$$\sigma_s = 0.05$$

$$\sigma_m = 0.08$$

$$I = 5$$

$$J_i = 10$$

$$t_{ij} = (0, 2, 4, 6, 8, 15, 25, 35, 45, 55) \text{ minutes every } i$$

$$T_{ij} = (1, 1, 1, 1, 1, 5, 5, 5, 5, 5) \text{ minutes every } i$$

$$T_{ij}^* = 10 \text{ minutes every } i \text{ and every } j$$

$B_{ij} = 10$ cpm every i and every j

$U_{ij} = 1$ every i and every j

The output C_{ij} and C_{ij}^* were analyzed with the computer program SAND (Appendix), as explained in the next subsection.

Results

The 15 sets of C_{ij} and C_{ij}^* together with the information

$(\lambda_1, \lambda_2, \lambda_3) = (0.30, 0.15, 0.05)/\text{min}$

$t_{ij} = (0, 2, 4, 6, 8, 15, 25, 35, 45, 55)$ minutes

$T_{ij} = (1, 1, 1, 1, 1, 5, 5, 5, 5, 5)$ minutes

$T_{ij}^* = 10$ minutes

$\tau = 0$ minute

were inputted to the computer program in succession, thereby accumulating historic $\tilde{\sigma}_m$ and $\tilde{\sigma}_s$ with associated degrees of freedom after each set for use in the next set. Set 1 was rerun after set 15 and then set 2 was rerun. These runs are designated 1' and 2' in Tables 1 through 6, and the slight differences from 1 and 2 noted stem from the change in historic $\tilde{\sigma}_m$ and $\tilde{\sigma}_s$. The historic estimates of percentage errors are graphed in Figure 1.

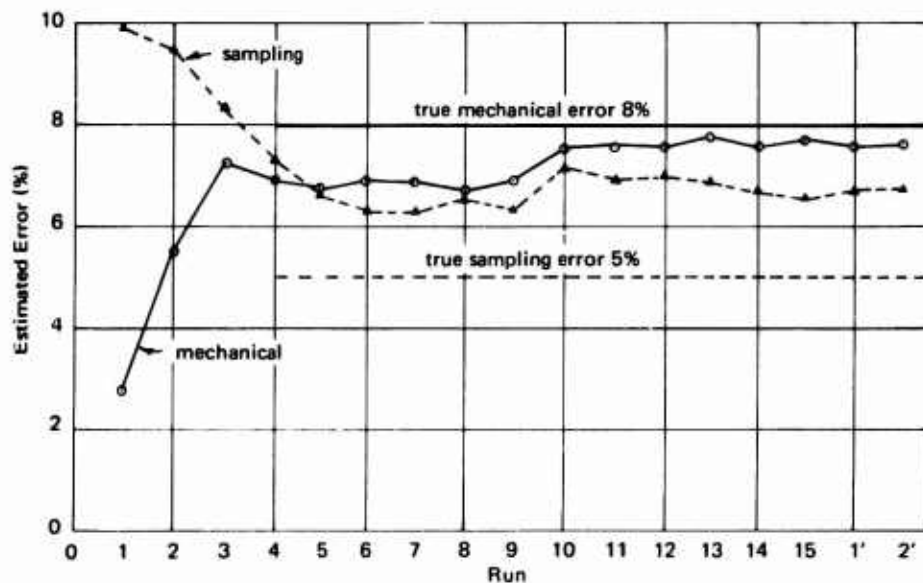


Figure 1. Estimated sampling and mechanical errors versus computer run number.

The total uncertainty in $\tilde{\mu}_0^0$ expressed as a variance appears in the last column of Table 1. The other columns contain the analysis of the total uncertainty into sampling, mechanical, and Poisson sources.

The 17 estimates of μ_0^0 , μ_1^0 , μ_2^0 , and μ_3^0 with 0.95 confidence intervals are contained in Table 2. It will be recalled that $\mu_0^0 = 10,000$, $\mu_1^0 = 8,000$, $\mu_2^0 = 1,500$, and $\mu_3^0 = 500$ cpm.

The running estimates (current and historical) of σ_m and σ_s are contained in Table 3. The degrees of freedom for 1' and 2' are admittedly wrong since the old data of 1 and 2 were used, giving really no new degrees of freedom.

In detail Table 4 shows the estimates of μ_0^0 , μ_1^0 , μ_2^0 , and μ_3^0 from each of the 75 samples. The reader will note the entries are wilder than those of Table 2, which showed the more stable weighted averages of five samples.

Uncertainty in estimating μ_0^0 from a single sample expressed as total variance is analyzed into mechanical and Poisson sources in Table 5 for each of the 75 samples.

For each of the 75 samples Table 6 displays S_i^2 and the product S_i^2 times the elements of the symmetric inverse matrix under the headings of variances and covariances.

Table 1. Analysis of Variance of Mean Count
Rate at Time $t = 0$ (All Isotopes)

Run	Component			Total
	Sampling	Mechanical	Poisson	
1	234492.	26212.	14726.	275432.
2	151186.	34926.	15107.	201221.
3	67015.	22903.	8934.	98853.
4	15476.	7010.	1553.	24041.
5	6284.	2416.	646.	9346.
6	43984.	20101.	6206.	70292.
7	67485.	37719.	8886.	114091.
8	148952.	70537.	16551.	236041.
9	38675.	14239.	4133.	57048.
10	566514.	291941.	96929.	955385.
11	36820.	19376.	4379.	60576.
12	136088.	43760.	11596.	191446.
13	48631.	27593.	8649.	84874.
14	23217.	8967.	1744.	33929.
15	29841.	15396.	5465.	50703.
1'	180106.	68296.	11192.	259595.
2'	119166.	65858.	19668.	204693.

Table 2. Mean Decay Rate at Time $t = 0$
and 0.95 Confidence Limits

Run	All Isotopes (cpm)		
	Lower Limit	Mean ($\tilde{\mu}_{..}^0$)	Upper Limit
1	9277.1	10734.2	12191.3
2	8476.0	9721.4	10966.9
3	8910.6	9783.6	10656.5
4	9981.6	10412.1	10842.6
5	9588.2	9856.6	10125.0
6	9884.8	10620.9	11357.0
7	8976.2	9914.0	10851.8
8	8759.3	10108.2	11457.0
9	9078.5	9741.6	10404.8
10	6461.8	9175.6	11889.4
11	9106.9	9790.2	10473.6
12	9254.6	10469.4	11684.2
13	9382.7	10191.6	11000.5
14	9662.2	10173.6	10685.0
15	8769.9	9395.1	10020.3
1'	9461.6	10876.2	12290.8
2'	8482.7	9738.9	10995.0
Run	First Isotope (cpm)		
	Lower Limit	Mean ($\tilde{\mu}_1^0$)	Upper Limit
1	7029.8	8442.4	9855.1
2	6192.8	7516.3	8839.9
3	6338.5	7525.1	8719.6
4	7908.2	8749.8	9591.4
5	7543.6	7918.1	8292.7
6	7798.9	8872.1	9945.3
7	7330.7	7805.8	8280.9
8	6886.9	8200.7	9514.5
9	6865.7	7474.0	8082.4
10	2678.6	7037.2	11395.8
11	6557.0	7564.6	8572.2
12	6460.0	8587.5	10715.0
13	7502.9	8204.4	8905.8
14	7036.4	8069.6	9102.7
15	6519.0	7394.9	8270.8
1'	7177.7	8577.6	9977.4
2'	6204.8	7538.0	8871.1

Continued

Table 2. Continued

Run	Second Isotope (cpm)		
	Lower Limit	Mean ($\tilde{\mu}_2^o$)	Upper Limit
1	1663.5	1788.9	1914.2
2	1180.2	1729.7	2279.1
3	1203.0	1754.9	2306.8
4	397.3	1158.1	1918.8
5	1248.1	1432.5	1616.9
6	621.6	1222.1	1822.6
7	906.2	1619.8	2333.5
8	984.4	1424.4	1864.5
9	1677.6	1794.2	1910.9
10	-110.2	1632.5	3375.3
11	1225.2	1718.5	2211.7
12	238.7	1354.0	2469.3
13	1241.7	1495.1	1748.6
14	713.4	1613.9	2514.4
15	1045.0	1524.1	2003.3
1'	1671.7	1791.7	1911.7
2'	1183.0	1725.0	2267.0
Run	Third Isotope (cpm)		
	Lower Limit	Mean ($\tilde{\mu}_3^o$)	Upper Limit
1	476.4	502.8	529.1
2	437.6	475.3	513.1
3	456.6	499.5	542.4
4	463.3	504.1	544.8
5	466.6	505.9	545.1
6	481.0	526.6	572.3
7	459.1	488.3	517.4
8	425.1	482.9	540.8
9	391.5	473.3	555.0
10	437.4	505.8	574.3
11	472.1	507.1	542.0
12	457.5	527.8	598.2
13	466.8	492.0	517.2
14	477.9	490.0	502.2
15	408.8	475.9	543.1
1'	471.4	506.8	542.2
2'	439.3	475.8	512.3

Table 3. Estimates of Current and Historic Mechanical and Sampling Errors

Run	Mechanical Error			Sampling Error		
	Current (%)	Historic (%)	D.F. ^{1/}	Current (%)	Historic (%)	D.F. ^{1/}
0	—	8.00	0	—	5.00	0
1	2.84	2.84	35	9.90	9.90	4
2	7.21	5.48	70	8.95	9.44	8
3	9.81	7.22	105	5.47	8.33	12
4	5.82	6.90	140	2.64	7.33	16
5	6.09	6.74	175	1.78	6.60	20
6	7.78	6.93	210	4.47	6.30	24
7	6.28	6.84	245	5.88	6.24	28
8	5.58	6.69	280	8.44	6.56	32
9	8.03	6.86	315	3.73	6.31	36
10	12.03	7.53	350	12.16	7.11	40
11	8.13	7.59	385	4.42	6.91	44
12	7.23	7.56	420	7.95	7.00	48
13	9.63	7.74	455	4.93	6.87	52
14	4.84	7.57	490	3.34	6.68	56
15	9.44	7.71	525	4.11	6.54	60
1'	4.88	7.56	560	8.49	6.68	64
2'	8.40	7.61	595	7.96	6.76	68

^{1/} D.F. = Degrees of freedom associated with updated historic estimate.

Table 4. Mean Decay Rates at Time $t = 0$

Run	Sample (i)	Isotope (cpm)			
		All ($\tilde{\mu}_0^\circ$)	First ($\tilde{\mu}_1^\circ$)	Second ($\tilde{\mu}_2^\circ$)	Third ($\tilde{\mu}_3^\circ$)
1	1	9075.5	6698.7	1859.0	517.7
	2	12219.9	9790.3	1901.1	528.4
	3	11225.4	8920.0	1813.9	491.4
	4	10018.4	7880.4	1663.9	473.9
	5	10774.7	8572.0	1701.5	501.1
2	1	9152.7	7013.6	1637.8	501.2
	2	10766.9	8815.5	1465.9	485.4
	3	10830.6	8453.6	1889.0	487.9
	4	8727.2	7021.8	1222.0	483.3
	5	9110.2	6327.0	2358.3	424.8
3	1	9923.6	8238.1	1157.8	527.6
	2	9565.3	6857.8	2215.4	492.0
	3	11121.8	9055.2	1521.8	544.7
	4	9249.4	7101.5	1693.7	454.1
	5	9366.8	6755.2	2121.8	489.7

Continued

Table 4. Continued

Run	Sample (i)	Isotope (cpm)			
		All ($\mu_{..}^{\circ}$)	First (μ_1°)	Second (μ_2°)	Third (μ_3°)
4	1	10597.1	8923.6	1160.2	507.2
	2	10006.6	7925.4	1602.2	478.9
	3	10483.6	9737.8	187.6	558.0
	4	10137.5	8493.7	1154.0	489.7
	5	10872.9	8647.2	1742.3	483.3
5	1	9694.7	7756.2	1440.8	497.6
	2	10051.1	8346.1	1180.6	524.3
	3	9980.2	7846.5	1583.2	550.4
	4	9565.7	7566.8	1514.8	483.9
	5	10019.6	8130.0	1417.2	472.2
6	1	10289.3	8260.2	1563.1	465.8
	2	10728.6	8634.0	1518.6	575.9
	3	9767.4	8194.1	1057.4	515.8
	4	11293.6	10248.6	510.2	534.7
	5	10852.0	8724.1	1596.6	531.2
7	1	9463.3	7684.7	1293.5	484.9
	2	10570.3	8039.8	2026.3	504.0
	3	9670.3	7307.0	1910.5	452.8
	4	9013.5	7779.7	727.2	506.6
	5	10775.2	8249.1	2026.4	499.6
8	1	8834.8	7204.9	1188.8	440.9
	2	11592.1	9376.7	1688.5	526.8
	3	9309.6	7009.9	1872.8	426.8
	4	10418.7	8956.2	954.8	507.6
	5	10558.5	8676.3	1365.4	516.7
9	1	9787.4	7681.8	1719.1	386.4
	2	10883.7	8397.0	1990.4	496.2
	3	9157.6	6869.5	1804.1	483.8
	4	9876.2	7476.4	1854.9	544.7
	5	9601.5	7340.8	1744.5	516.2
10	1	10068.6	8732.8	786.8	548.9
	2	2386.6	-4007.7	6037.1	357.2
	3	9539.8	7265.5	1781.0	493.2
	4	8612.3	6462.7	1669.7	479.7
	5	10428.2	8743.2	1147.4	537.4
11	1	9303.7	7232.3	1586.9	484.5
	2	9911.3	7996.5	1411.6	503.0
	3	10604.2	8593.1	1488.7	522.3
	4	9275.1	6340.7	2454.2	480.1
	5	9824.4	7339.7	1932.4	552.2

Continued

Table 4. Continued

Run	Sample (i)	Isotope (cpm)			
		All ($\tilde{\mu}_{..}^o$)	First ($\tilde{\mu}_1^o$)	Second ($\tilde{\mu}_2^o$)	Third ($\tilde{\mu}_3^o$)
12	1	12061.1	11422.0	23.0	616.0
	2	10242.7	8069.6	1663.2	509.8
	3	9720.2	8114.4	1080.6	525.1
	4	10308.4	7497.9	2349.3	461.1
	5	9761.9	7316.9	1936.7	508.2
13	1	9169.9	7413.9	1253.7	502.2
	2	10490.0	8786.0	1254.7	449.1
	3	10880.1	8763.3	1621.2	495.5
	4	10218.1	8104.0	1613.8	500.2
	5	10274.7	8110.2	1663.9	500.5
14	1	9786.2	8217.4	1074.2	494.4
	2	9683.3	7503.2	1700.3	479.7
	3	10645.6	9381.8	781.9	481.8
	4	10361.3	7324.9	2531.9	504.3
	5	10349.2	7743.9	2115.4	489.9
15	1	8783.8	6940.2	1377.3	466.1
	2	9021.0	6618.1	1957.9	444.9
	3	9862.1	8028.2	1282.4	551.4
	4	9866.4	8366.9	987.3	512.0
	5	9640.6	7392.1	1831.3	417.1
1'	1	9326.7	7014.7	1785.1	526.9
	2	12396.6	10078.2	1775.5	542.8
	3	11184.0	8774.0	1927.7	482.2
	4	10081.1	7804.9	1801.4	474.7
	5	10853.6	8677.5	1672.6	503.4
2'	1	9159.6	7017.2	1641.4	500.9
	2	10791.1	8870.5	1431.1	489.4
	3	10835.2	8457.6	1890.0	487.5
	4	8727.7	7007.6	1238.0	482.1
	5	9129.1	6370.9	2330.7	427.4

Table 5. Analysis of Variance of Mean Decay Rate at Time $t = 0$
(All Isotopes - Fixed Sample)

Run	Sample (i)	Weight (V_i)	Component of Variance		Total Variance
			Mechanical	Poisson	
1	1	0.1784	263289.	103927.	367217.
	2	0.2106	88950.	49163.	138114.
	3	0.2142	71187.	45703.	116891.
	4	0.1829	187432.	142537.	329970.
	5	0.2136	78093.	42290.	120384.
2	1	0.1834	234844.	149464.	384308.
	2	0.1928	233754.	90652.	324406.
	3	0.2093	181006.	51120.	232127.
	4	0.1990	234582.	53492.	288075.
	5	0.2153	113077.	89236.	202314.
3	1	0.2096	212042.	69042.	281085.
	2	0.2214	184160.	46395.	230555.
	3	0.1599	391622.	183624.	575247.
	4	0.2091	183676.	99464.	283140.
	5	0.1997	238789.	89202.	327992.
4	1	0.2076	231941.	60566.	292507.
	2	0.2114	254415.	22040.	276455.
	3	0.2081	243324.	47046.	290371.
	4	0.1894	320292.	56082.	376374.
	5	0.1833	286201.	122176.	408378.
5	1	0.2254	91493.	47235.	138729.
	2	0.1823	235998.	35760.	271759.
	3	0.2068	162977.	26366.	189344.
	4	0.1924	172516.	62571.	235088.
	5	0.1929	186934.	46729.	233664.
6	1	0.1604	331838.	127761.	459599.
	2	0.1835	297079.	48237.	345316.
	3	0.2012	180200.	95300.	275501.
	4	0.2267	151903.	42326.	194230.
	5	0.2279	153969.	36814.	190784.
7	1	0.1589	387264.	67945.	455209.
	2	0.1648	317444.	107897.	425341.
	3	0.2380	159895.	16860.	176756.
	4	0.2019	200948.	75684.	276633.
	5	0.2361	150803.	30359.	181162.

Continued

Table 5. Continued

Run	Sample (i)	Weight (V_i)	Component of Variance		Total Variance
			Mechanical	Poisson	
8	1	0.2135	181379.	45646.	227026.
	2	0.1840	307707.	26350.	334058.
	3	0.2005	215719.	54452.	270172.
	4	0.1569	383884.	83341.	467225.
	5	0.2448	95522.	46152.	141674.
9	1	0.2914	34774.	7777.	42551.
	2	0.1112	549551.	173906.	723458.
	3	0.2207	146099.	31088.	177187.
	4	0.1502	294614.	142978.	437592.
	5	0.2262	137816.	25790.	163606.
10	1	0.2698	116786.	54891.	171678.
	2	0.0682	1752976.	184104.	1937081.
	3	0.1959	354252.	43133.	397385.
	4	0.2380	162209.	89331.	251540.
	5	0.2278	189679.	92461.	282140.
11	1	0.2535	102776.	53193.	155969.
	2	0.2177	232965.	23992.	256957.
	3	0.2088	244970.	42622.	287593.
	4	0.1527	481337.	80013.	561350.
	5	0.1670	367288.	106687.	473975.
12	1	0.2192	149618.	23907.	173525.
	2	0.2579	47669.	18974.	66643.
	3	0.1851	253867.	50431.	304299.
	4	0.1596	321370.	117524.	438895.
	5	0.1779	265270.	73032.	338302.
13	1	0.2122	264342.	64764.	329106.
	2	0.1501	536774.	131538.	668313.
	3	0.2172	210208.	100169.	310378.
	4	0.2188	234266.	70318.	304584.
	5	0.2014	280819.	92317.	373136.
14	1	0.2136	136544.	36041.	172586.
	2	0.1808	265517.	22411.	287929.
	3	0.2118	131970.	46039.	178010.
	4	0.1886	223318.	33582.	256900.
	5	0.2049	167787.	31691.	199478.

Continued

Table 5. Continued

Run	Sample (i)	Weight (V_i)	Component of Variance		Total Variance
			Mechanical	Poisson	
15	1	0.2183	159831.	63264.	223095.
	2	0.2219	170052.	43352.	213405.
	3	0.2123	175151.	64917.	240068.
	4	0.1417	400372.	147496.	547869.
	5	0.2057	186634.	73194.	259828.
1'	1	0.1738	336522.	29850.	366372.
	2	0.2167	159610.	29845.	189455.
	3	0.2240	131648.	34580.	166229.
	4	0.1558	393045.	76615.	469660.
	5	0.2294	134413.	15233.	149647.
2'	1	0.1688	331167.	125479.	456646.
	2	0.1887	287619.	75461.	363080.
	3	0.2185	206020.	48241.	254262.
	4	0.2042	265972.	36648.	302621.
	5	0.2195	160941.	90226.	251168.

Table 6. Variances and Covariances of Fitted Parameters and Weighted Variance of Errors

Run	Sample (i)	S_i^2	$\text{Var}(\tilde{\mu}_{1i}^0)$	$\text{Cov}(\tilde{\mu}_{1i}^0, \tilde{\mu}_{2i}^0)$	$\text{Var}(\tilde{\mu}_{2i}^0)$	$\text{Cov}(\tilde{\mu}_{1i}^0, \tilde{\mu}_{3i}^0)$	$\text{Cov}(\tilde{\mu}_{2i}^0, \tilde{\mu}_{3i}^0)$	$\text{Var}(\tilde{\mu}_{3i}^0)$
1	1	33.7677	1088353.	-547692.	355914.	31838.9	-24505.3	3665.85
	2	8.9005	361420.	-168028.	106941.	9613.9	-7221.5	1023.67
	3	7.4815	307503.	-142235.	89047.	7856.0	-5880.7	859.79
	4	25.7002	902410.	-432518.	277844.	25136.9	-19175.5	2829.26
	5	8.7215	322766.	-152531.	97494.	8803.1	-6706.9	993.14
2	1	15.7835	984075.	-440019.	267162.	20284.6	-14742.5	2026.01
	2	10.0429	770050.	-322227.	188685.	14549.8	-10147.0	1319.44
	3	6.9821	560264.	-238000.	140816.	10301.7	-7243.0	931.07
	4	12.6283	730381.	-324027.	195511.	15672.6	-11345.9	1585.19
	5	6.8547	516231.	-230523.	140873.	9695.9	-7022.8	909.78
3	1	7.9947	671378.	-280917.	163144.	12050.1	-8407.7	1112.25
	2	6.1783	575352.	-250694.	150135.	9693.8	-6904.2	876.75
	3	14.1101	1351449.	-556948.	321629.	22697.8	-15661.7	1992.41
	4	7.5178	685747.	-291100.	171532.	11758.0	-8252.0	1050.84
	5	9.1600	821349.	-359011.	215288.	14131.1	-10090.2	1294.50
4	1	7.6392	678490.	-276633.	158790.	11874.1	-8150.5	1047.65
	2	7.7014	663457.	-279426.	163754.	11691.6	-8167.2	1047.96
	3	9.1120	663059.	-266015.	149834.	13022.8	-8877.8	1218.81
	4	10.6461	880863.	-362356.	208912.	15929.1	-10988.7	1430.11
	5	9.4241	951666.	-390139.	225884.	15621.2	-10727.4	1319.60

Continued

Table 6. Continued

Run	Sample (i)	S_i^2	$\text{Var}(\tilde{\mu}_{1i}^0)$	$\text{Cov}(\tilde{\mu}_{1i}^0, \tilde{\mu}_{2i}^0)$	$\text{Var}(\tilde{\mu}_{2i}^0)$	$\text{Cov}(\tilde{\mu}_{1i}^0, \tilde{\mu}_{3i}^0)$	$\text{Cov}(\tilde{\mu}_{2i}^0, \tilde{\mu}_{3i}^0)$	$\text{Var}(\tilde{\mu}_{3i}^0)$
5	1	4.1177	336106.	-142734.	83898.	6113.5	-4298.7	564.15
	2	8.0103	649626.	-272321.	158516.	11879.0	-8299.3	1099.80
	3	5.9976	468293.	-202368.	119980.	8628.0	-6133.3	817.52
	4	6.8728	570134.	-242407.	142731.	10276.7	-7229.9	943.06
	5	6.3566	551974.	-229214.	133264.	9759.9	-6766.1	867.02
6	1	11.3843	1076076.	-443074.	256718.	18287.4	-12600.9	1580.39
	2	9.9056	836648.	-354698.	207805.	14924.2	-10484.4	1380.22
	3	8.4085	658478.	-275988.	160455.	12259.9	-8558.1	1140.48
	4	4.9354	433352.	-169978.	95081.	7748.5	-5211.8	680.10
	5	4.9221	452643.	-188424.	109550.	7764.2	-5388.9	688.67
7	1	13.5076	1095800.	-462616.	270839.	19991.4	-14010.9	1842.05
	2	10.5730	1024604.	-432839.	254656.	17074.9	-11945.5	1500.08
	3	4.7374	428202.	-182058.	107617.	7378.0	-5177.4	652.50
	4	9.6285	667162.	-281983.	164304.	13213.0	-9285.8	1278.67
	5	4.3012	432075.	-180876.	105926.	7068.2	-4917.7	611.36
8	1	7.1099	546340.	-230889.	135398.	10240.6	-7180.6	946.89
	2	7.9423	777163.	-317808.	183312.	12913.4	-8867.8	1108.59
	3	7.2506	654445.	-278396.	164788.	11311.4	-7945.9	1001.18
	4	13.2589	1084811.	-442860.	253984.	19774.3	-13588.1	1777.11
	5	3.8421	334719.	-138881.	80555.	5922.6	-4107.3	531.38

Continued

Table 6. Continued

Run	Sample (i)	S_i^2	$\text{Var}(\tilde{\mu}_{1i}^0)$	$\text{Cov}(\tilde{\mu}_{1i}^0, \tilde{\mu}_{2i}^0)$	$\text{Var}(\tilde{\mu}_{2i}^0)$	$\text{Cov}(\tilde{\mu}_{1i}^0, \tilde{\mu}_{3i}^0)$	$\text{Cov}(\tilde{\mu}_{2i}^0, \tilde{\mu}_{3i}^0)$	$\text{Var}(\tilde{\mu}_{3i}^0)$
9	1	.9526	98001.	-39880.	23071.	1606.0	-1097.6	132.68
	2	16.8266	1711478.	-713580.	416584.	27975.2	-19399.2	2405.10
	3	5.3747	441866.	-192529.	115085.	7951.1	-5675.6	743.12
	4	13.1918	1003278.	-476555.	284401.	19572.4	-13983.4	1845.22
	5	4.8999	405877.	-175909.	104668.	7282.5	-5183.8	681.68
10	1	4.9398	404175.	-166700.	95667.	7388.0	-5112.3	685.19
	2	225.8252	10914405.	-7580501.	6129041.	235380.4	-224993.0	33863.57
	3	10.4559	970684.	-414654.	244812.	16441.5	-11585.1	1484.35
	4	7.7211	631253.	-276167.	165119.	11307.1	-8089.7	1066.87
	5	7.3557	663043.	-273143.	157204.	11543.4	-7973.4	1039.11
11	1	4.2321	379429.	-161501.	95056.	6557.5	-4612.3	595.98
	2	6.7440	612012.	-255460.	148503.	10530.1	-7319.9	942.38
	3	6.9893	676727.	-279230.	161340.	11296.4	-7797.7	990.28
	4	14.4665	1418204.	-624489.	376812.	23291.8	-16685.0	2098.47
	5	13.2205	1184756.	-515962.	307523.	20445.9	-14590.0	1908.90
12	1	4.3711	382297.	-147622.	81293.	6864.9	-4579.6	610.62
	2	1.6247	158958.	-66436.	38717.	2641.6	-1837.3	233.03
	3	8.5209	725537.	-302835.	175416.	12898.1	-8984.9	1189.09
	4	9.3433	1051403.	-441957.	260140.	16262.6	-11308.7	1359.62
	5	8.5873	829790.	-355677.	210509.	13807.4	-9750.7	1244.24

Continued

Table 6. Continued

Run	Sample (i)	S_i^2	$\text{Var}(\hat{\mu}_{1i}^0)$	$\text{Cov}(\hat{\mu}_{1i}^0, \hat{\mu}_{2i}^0)$	$\text{Var}(\hat{\mu}_{2i}^0)$	$\text{Cov}(\hat{\mu}_{1i}^0, \hat{\mu}_{3i}^0)$	$\text{Cov}(\hat{\mu}_{2i}^0, \hat{\mu}_{3i}^0)$	$\text{Var}(\hat{\mu}_{3i}^0)$
13	1	9.4799	798281.	-338593.	198208.	14267.5	-10029.2	1327.64
	2	13.9405	1501639.	-593240.	335503.	23963.1	-16125.5	1974.99
	3	6.7058	717294.	-291089.	167060.	11412.5	-7791.3	958.99
	4	7.1296	720601.	-298834.	173377.	11777.7	-8153.5	1027.21
	5	8.7368	883540.	-366731.	212941.	14402.9	-9970.6	1251.76
14	1	4.5290	404382.	-166217.	95616.	7069.8	-4875.8	633.40
	2	7.3829	692988.	-292244.	171356.	11682.3	-8162.8	1035.56
	3	4.1221	396627.	-155302.	87003.	6706.7	-4499.4	570.62
	4	5.7695	630653.	-270713.	161018.	9832.6	-6929.8	849.09
	5	4.5119	478815.	-201438.	118270.	7612.4	-5304.4	653.29
15	1	6.3600	542794.	-231042.	135822.	9589.3	-6748.4	882.08
	2	5.5230	523591.	-224797.	133486.	8732.7	-6158.7	773.23
	3	6.8174	581808.	-246383.	143916.	10329.9	-7256.2	963.51
	4	14.2975	1282782.	-526465.	302110.	22368.3	-15423.1	2016.90
	5	5.5942	608525.	-250540.	145592.	9600.7	-6605.3	800.21
1'	1	10.8113	917516.	-400345.	238808.	16141.1	-11530.7	1517.57
	2	3.6541	430101.	-171473.	97516.	6555.8	-4431.1	535.70
	3	3.3346	383439.	-155448.	89450.	5895.8	-4019.5	483.36
	4	10.6943	1113893.	-463465.	270107.	17989.2	-12468.2	1548.71
	5	3.3043	348635.	-142586.	82197.	5598.9	-3843.9	477.20
2'	1	13.1636	1129638.	-487822.	289272.	19863.4	-14095.5	1844.93
	2	7.9225	832555.	-335373.	191553.	13433.9	-9140.6	1132.78
	3	5.3394	593307.	-243107.	140574.	9274.7	-6365.2	775.64
	4	9.2840	740481.	-316722.	186362.	13533.2	-9557.8	1272.78
	5	5.8642	618130.	-266301.	159016.	9827.7	-6940.0	848.61

Discussion of Results

Fifteen runs on different simulated samples of data with parameters $\mu_{..}^o = \mu_1^o + \mu_2^o + \mu_3^o$ equal to

$$10,000 = 8,000 + 1,500 + 500$$

have been analyzed by means of the FORTRAN program. For each run 0.95 confidence intervals for $\mu_{..}^o$, μ_1^o , μ_2^o , and μ_3^o were printed as outputs (see Table 2). If the procedure were perfect, in the long run 5% of the intervals would fail to bracket (contain) the parameter represented. Five percent of fifteen is 0.75. Thus a priori the expected number of misses per parameter is 0.75 and for all four parameters three misses. Actually, two misses were experienced, μ_2^o at run 1 and μ_2^o at run 9. If the procedure is imperfect with reference to confidence intervals, a large number of additional runs would be needed to "prove" it statistically.

The data generator introduced mechanical error with parameter

$$\sigma_m = 0.08$$

From the 15 runs with their available 525 degrees of freedom for estimating σ_m , the final estimate was:

$$\tilde{\sigma}_m = 0.0771$$

Is the procedure biased downward? The difference -0.0029 is not statistically significant at the 0.95 level. Thus more runs would be needed to prove that the procedure is imperfect for estimating σ_m . Even if the procedure is imperfect, the bias appears to be small. The authors anticipated a small upward bias rather than the downward bias actually observed, because the variance of the sum of the mechanical and Poisson components should be slightly more than the sum of the two components, due to a small positive covariance. This covariance stems from the slight tendency for large Poisson errors to be associated with large mechanical error since σ_m is on a percentage basis. The procedure ignores this tendency by partitioning as though the two sources were independent. The procedure should have a tendency to lead to a divisor too small in the proportioning for $\tilde{\sigma}_m$ and thus give a $\tilde{\sigma}_m$ too large.

The data generator introduced sampling error with parameter

$$\sigma_s = 0.05$$

From the 15 runs with their available 60 degrees of freedom for estimating σ_s , the final estimate $\tilde{\sigma}_s = 0.0654$ misses the parametric value by a disquieting amount. While the physicist may not be concerned with a bias of 1-1/2%, this is statistically significant at the 0.99 level, from which it is concluded that the procedure estimates σ_s with an upward bias. Why? Some of it can stem from the fact that the procedure treats sampling, Poisson, and mechanical error sources as independent, whereas because of the percentage concept there are actually three small positive covariances between these three sources. In the proportioning for $\tilde{\sigma}_s$ these were not included in the divisor – which should result in a tendency to bias $\tilde{\sigma}_s$ upward.

It is noted that σ_s is estimated at the last step of the computation. It is therefore suspected that the accumulated effects of all the slight imperfections of the procedure finally appear in $\tilde{\sigma}_s$. Two of the approximations suspected of contributing a large part of the imperfections are (1) the use of estimated variances instead of true variances in the weighting and (2) the inefficient estimation of Poisson parameters by the partitioning of each regression error. (A discussion of the efficiency of estimating Poisson parameters can be found in another report.*)

Imprecise as the procedure may be it is believed to be a much better approach than the customary assumption that all error is Poisson distributed. It is hoped that physicists will use this procedure and that statisticians will attempt to improve it.

FINDINGS

A procedure implemented as the SAND computer program is now available for use by nuclear physicists. Given a multiple isotope source and a series of observations on some samples of the source, this FORTRAN computer program will estimate the strength of each isotope; place confidence limits thereon; break the uncertainty into Poisson error, sampling error, and residual (called mechanical) error; supply an estimate of the standard deviation of sampling error; and supply an estimate of the standard deviation of mechanical error.

* U. S. Naval Civil Engineering Laboratory. Technical Report R-519: Efficiency of two estimates for a Poisson distribution, by W. L. Wilcoxon and M. L. Eaton. Port Hueneme, Calif., Apr 1967.

CONCLUSIONS

The procedure developed in this report constitutes a considerable improvement over the customary technique, whereby overoptimistic confidence intervals have been based on only Poisson variation. Even though the procedure contains statistical imperfections that would require considerably more research to remove, it does a reasonably good job of breaking total variation into its three sources.

RECOMMENDATION

It is suggested that nuclear physicists use the procedure described in this report for the statistical analysis of counting data.

Appendix

SAND, A FORTRAN II-D COMPUTER PROGRAM FOR THE STATISTICAL ANALYSIS OF NUCLEAR DATA

The procedure described within the body of this report is implemented by the computer program SAND on an IBM 1620 with an IBM 1311 Disk Drive. A minimum 1620 (20K memory) with no special features but a disk is required. The programming language is the disk version of FORTRAN II; however, the disk requirement may be replaced by magnetic tape by changing the FETCH and RECORD statements in the following subroutines: FANDW, INPUT2, and BCKGND.

All input to the program is from the card reader, and all output is to the card punch for off-line listing on an IBM 407. The card I/O statements may be replaced by tape I/O through a simple sifting program.

The program for the IBM 1620 is limited to a maximum of 25 samples per run.

The format of the input for each run, the card deck setup (Figure 2), a sample input with corresponding output (Figure 3), and the listing of the FORTRAN II-D source program (Figure 4) follow.

FORMAT OF INPUT FOR EACH RUN

Card 1. Title card; FORMAT (80H)

Card 2. $I, L, \tau, t_o, \tilde{\sigma}_{mo}, f_o, \tilde{\sigma}_{so}, d, \tilde{\sigma}_{mr}$; FORMAT (2I5, 7E10.0)

where I = numbers of samples ($I \leq 25$)

L = number of isotopes ($L \leq 5$)

τ = detector deadtime in min/count

t_o = historic time at $t = 0$ (minutes)

$\tilde{\sigma}_{mo}$ = historic estimate of mechanical error (percent)

f_o = degrees of freedom associated with $\tilde{\sigma}_{mo}$. (If $f_o = 0$, then this is a starting estimate used in solving the nonlinear equation in $\tilde{\sigma}_m$.)

$\tilde{\sigma}_{so}$ = historic estimate of sampling error (percent)

d = degrees of freedom associated with $\tilde{\sigma}_{so}$. (If $d = 0$, then this is a starting estimate used in solving the nonlinear equation in $\tilde{\sigma}_s$.)

$\tilde{\sigma}_{mr}$ = partially updated $\tilde{\sigma}_{mo}$ used in restarting program after interruption (normally blank)

Card 3. $\lambda_1, \lambda_2, \dots, \lambda_L$; FORMAT (5E15.8)

where λ_ℓ = decay constant for isotope ℓ ($\ell = 1, 2, \dots, L \leq 5$) in reciprocal minutes

Card 4. J_1, J_2, \dots, J_I ; FORMAT (15I5)

where J_i = number of observations for sample i ($i = 1, 2, \dots, I \leq 25$)

Observation Cards. $T_{ij}, C_{ij}, T_{ij}^*, C_{ij}^*, t_{ij}$; FORMAT (5E15.8)

where the subscript j varies the most rapid; $j = 1, 2, 3, \dots, J_i$ for $i = 1, 2, 3, \dots, I$

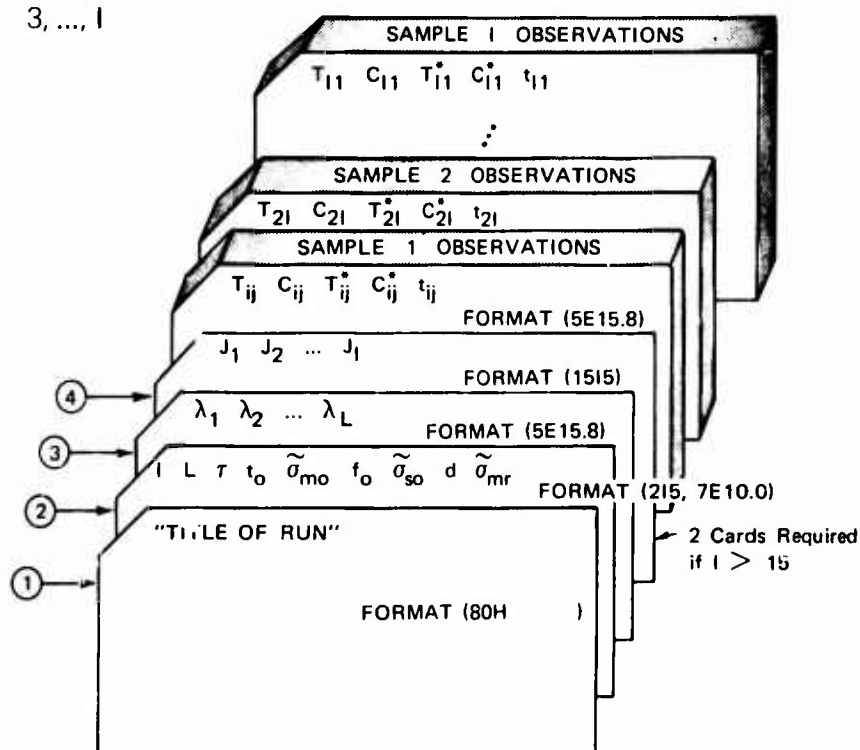


Figure 2. Deck setup for each run.

Sample Input

CONTINUED

Figure 3. Continued.

1.	3209.	10.	85.	4.
1.	2049.	10.	99.	6.
1.	1295.	10.	101.	8.
5.	1665.	10.	97.	15.
5.	839.	10.	100.	25.
5.	393.	10.	94.	35.
5.	338.	10.	113.	45.
5.	207.	10.	102.	55.
1.	9241.	10.	93.	0.
1.	6100.	10.	127.	2.
1.	3528.	10.	89.	4.
1.	1941.	10.	95.	6.
1.	1298.	10.	98.	8.
5.	1618.	10.	92.	15.
5.	827.	10.	91.	25.
5.	463.	10.	100.	35.
5.	283.	10.	112.	45.
5.	204.	10.	81.	55.
1.	8935.	10.	87.	0.
1.	6093.	10.	103.	2.
1.	3483.	10.	106.	4.
1.	2383.	10.	100.	6.
1.	1401.	10.	109.	8.
5.	2037.	10.	108.	15.
5.	830.	10.	100.	25.
5.	509.	10.	99.	35.
5.	292.	10.	109.	45.
5.	216.	10.	104.	55.

ZZZZ FND OF JOR

CONTINUED

Figure 3. Continued.

Sample Output

1 STATISTICAL ANALYSIS OF NUCLEAR DATA (SAND)

0 SIMULATED DATA FOR RUN NUMREP 6
 NUMBER OF SAMPLES 5
 NUMBER OF ISOTOPES 3
 HISTORIC TIME AT T=0 0.00 MIN
 DETECTOR DEADTIME 0.00 MIN/COUNT
 DECAY CONSTANTS .30 1/MIN
 .15
 .05

OBSERVATIONS PER SAMPLE

HISTORIC MECHANICAL ERROR 6.74 PERCENT
 DEGREES OF FREEDOM 175
 HISTORIC SAMPLING ERROR 6.60 PERCENT
 DEGREES OF FREEDOM 20
 TOLERANCE ON ERROR .005 PERCENT
 MAXIMUM ITERATIONS ALLOWED 25

0
 ITERATION MECHANICAL ERROR HISTORIC
 1 CURRENT 9.98 7.37
 2 7.98 6.96
 3 7.80 6.92
 4 7.78 6.92
 5 7.78 6.92

CONTINUED

Figure 3. Continued.

0	ITERATION	SAMPLING ERROR CURRENT	HISTORIC
1		4.54	6.30
2		4.47	6.29
3		4.47	6.29
0	MEAN DECAY RATE AT TIME T=0 (ALL ISOTOPES)		
	ANALYSIS OF VARIANCE		
	COMPONENT	VARIANCE	
	SAMPLING		43952.
	MECHANICAL		20128.
	POISSON		6221.
	TOTAL		70303.
0	ISOTOPE	DECAY RATE (COUNTS/MIN) AT TIME T=0 .95 L.C.L.	MEAN RATE .95 U.C.L.
	ALL	9884.8	10620.9
	1	7799.0	8872.2
	2	621.5	1222.0
	3	481.0	526.6
0	UPDATED HISTORIC MECHANICAL ERROR	6.92 PERCENT	
	DEGREES OF FREEDOM	210	
	UPDATED HISTORIC SAMPLING ERROR	6.29 PERCENT	
	DEGREES OF FREEDOM	24	

CONTINUED

Figure 3. Continued.

SAMPLE	ALL	ISOTOPE			
		1	2	3	4
1	10289.6	8260.9	1562.8	465.9	
2	10728.6	8634.1	1518.5	575.9	
3	9767.2	8193.8	1057.6	515.8	
4	11293.7	10248.7	510.2	534.7	
5	10851.9	8724.0	1596.7	531.2	
		MEAN DECAY RATE AT TIME T=0			
		(ALL ISOTOPES - FIXED SAMPLE)			
		ANALYSIS OF VARIANCE			
		MECHANICAL	POISSON	TOTAL	
1	.1604	331558.	127674.	459233.	
2	.1835	296964.	48276.	345240.	
3	.2012	180034.	95420.	275454.	
4	.2267	151826.	42334.	194161.	
5	.2279	153900.	36837.	190739.	

NOTE: The slight discrepancies between the values in this sample output and those given in Tables 1 - 5 are the result of using historic mechanical and sampling errors truncated to three digits as inputs.

Figure 4. Program listing.

```

ZZJOB
ZZDUP
*DELETSAND
ZZFOR
*LDISKSAND
C ANALYSIS OF COUNTING DATA FROM SAMPLES OF RADIOACTIVE MATERIALS
C W. L. WILCOXSON
C *
C *
C *
C AMDA(L) = DECAY CONSTANT (1/MINUTES) FOR THE L'ITH ISOTOPE
C AVMECH = ABSOLUTE MECHANICAL VARIANCE
C AVMIJ = FRACTION OF SQUARED ERROR AT POINT IJ ATTRIBUTED
C TO MECHANICAL CAUSES
C AVM(I) = ABSOLUTE VARIANCE DUE TO MECHANICAL CAUSES
C AVPAM = ABSOLUTE VARIANCE DUE TO POISSON AND MECHANICAL
C AVPIJ = FRACTION OF SQUARED ERROR AT POINT IJ ATTRIBUTED
C TO POISSON PHENOMENA
C AVP(I) = ABSOLUTE VARIANCE DUE TO POISSON PROCESSES
C AVPOSN = ABSOLUTE VARIANCE DUE TO POISSON PROCESS
C AVSAMP = ABSOLUTE VARIANCE DUE TO SAMPLING
C A(L1,L2) = ELEMENT OF LEAST SQUARES MATRIX
C C = COUNTS OBSERVED DURING PERIOD T
C CSTAR = BACKGROUND COUNTS OBSERVED DURING PERIOD TSTAR
C DELTA = MAXIMUM ABSOLUTE RELATIVE ERROR ALLOWED
C DF = DEGREES OF FREEDOM
C DFMECH = DEGREES OF FREEDOM FOR HISTORIC MECHANICAL ERROR

```

CONTINUED

Figure 4. Continued.

```

C      DFSAMP      = DEGREES OF FREEDOM FOR HISTORIC SAMPLING ERROR      0028
C      EMECH       = MECHANICAL ERROR (PERCENT)                        0029
C      EMUDT       = SUM(EMU(I,L)*X(L),L=1,LMAX),                      0030
C      EMUQ        = SUBSCRIPTS I, J, AND K ARE IMPLIED.              0031
C      EMU(I,L)    = LAST SUM(V(I)*SEMU(I),I=1,IMAX)                  0032
C      EN          = EXPECTED COUNTS PER MINUTE AT TIME=TO ATTRIBUTED  0033
C      ENSTAF      = TO ISOTOPE L IN SAMPLE I. (FITTED PARAMETER)    0034
C      ESAMP       = (C/T)/(1.-TAU*(C/T))                             0035
C      I           = CSTAR/TSTAR                                     0036
C      IJ          = SAMPLING ERROR (PERCENT)                         0037
C      IMAX        = SAMPLE INDEX                                     0038
C      IF          = SINGLE SUBSCRIPT EQUIVALENT TO DOUBLE SCRIPT I,J. 0039
C      ISAMP       = NO. SAMPLES OF RADIOACTIVE MATERIAL              0040
C      J           = 1 FOR K=1, 2 FOR K GREATER THAN 1                0041
C      JMAX(I)     = 1 IF IMAX=1, 2 IF IMAX IS GREATER THAN 1         0042
C      JMAX(I)     = OBSERVATION INDEX                                0043
C      K           = JMAX(I) FOR SOME I                               0044
C      KMAX        = NO. OF OBSERVATIONS OF THE I' TH SAMPLE         0045
C      L           = ITERATION INDEX                                  0046
C      L1,L2       = MAXIMUM NUMBER OF ITERATIONS ALLOWED            0047
C      LMAX        = ISOTOPE INDEX                                    0048
C      LMAX1       = DUMMY INDEXES                                    0049
C      N           = NO. OF ISOTOPES IN THE SOURCE SOLUTION          0050
C      S2          = LMAX+1                                           0051
C      SEMU(I)     = 1 OR 2, A CONTROL VARIABLE                       0052
C      SEMU(I)     = TOTAL VARIANCE DUE TO MECHANICAL AND POISSON     0053
C      SEMU(I)     = SUM(EMU(I,L),L=1,LMAX)                           0054

```

CONTINUED

Figure 4. Continued.

C	STUT	=	STUDENT'S T FOR IMAX-1 DEGREES OF FREEDOM	0055
C			TO GIVE 95 PERCENT CONFIDENCE LIMITS.	0056
C	SUVMX	=	SUM(A(L1,L2),L1=1,LVAX),L2=1,LMAX)	0057
C	T	=	LENGTH OF TIME (MINUTES) FOR OBSERVATION OF SAMPLE	0058
C	TAU	=	DETECTOR DEADTIME (MINUTES)	0059
C	TEMP,TEMP1....	=	TEMPORARY STORAGE	0060
C	TIME	=	TIME AT BEGINNING OF TIME INTERVAL T	0061
C	TO	=	HISTORIC TIME REFERENCE	0062
C	TSTAR	=	LENGTH OF BACKGROUND OBSERVATION (MINUTES)	0063
C	U	=	NUMBER OF TIMES BACKGROUND OBSERVATION IS USED	0064
C	VEMUO	=	VARIANCE OF FMUO	0065
C	VMECH	=	MECHANICAL COMPONENT OF VARIANCE (PROPORTIONAL)	0066
C	VMECHH	=	HISTORIC MECHANICAL VARIANCE	0067
C	VMECHW	=	CURRENT VALUE OF VMECH USED FOR WEIGHTING	0068
C	VSAMP	=	SAMPLING COMPONENT OF VARIANCE (PROPORTIONAL)	0069
C	VSAMPH	=	HISTORIC SAMPLING VARIANCE	0070
C	VSEMU(I)	=	VARIANCE OF SEMU(I)	0071
C	V(I)	=	WEIGHT USED TO DETERMINE THE SAMPLING COMPONENT	0072
C			OF VARIANCE. (SUBSCRIPT K IS IMPLIED)	0073
C	W	=	WEIGHT TO DETERMINE MECHANICAL COMPONENT	0074
C			OF VARIANCE (SUBSCRIPTS I,J,K ARE IMPLIED).	0075
C	X(L)	=	INDEPENDENT VARIABLE IN THE LEAST SQUARES FIT	0076
C	X(LMAX1)	=	DEPENDENT VARIABLE IN THE LEAST SQUARES FIT	0077
C	*			0078
C			SENSE SWITCH 1 FOR MAJOR STATISTICS OUTPUT	0079
C			SENSE SWITCH 2 ON FOR OUTPUT OF TRANSFORMED DATA	0080
C			SENSE SWITCH 3 ON FOR OUTPUT OF WEIGHTS	0081

CONTINUED

Figure 4. Continued.

```

C      SENSE SWITCH 4 ON IF DIFFERENT BACKGROUND FOR EACH OBSERVATION
C      SORT RESULTANT OUTPUT ON CC 2 AND LIST ON 407
C      *
C      *
C      *
C      DEFINE DISK(20,2000)
C      DIMENSION X(5),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
1  AMDA(5),V(25),AVM(25),AVP(25)
C      COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
1  A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2  AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMECH,EMUDT,EMU0,ESAMP,I,IJ,
3  IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAXI,S2,SUMMX,TAU,TO,VEMU0,
4  VMECH,VMECHW,VSAMP
C      COMMON VNECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
C      FMECH=0.
C      DFMECH=0.
C      ESAMP=0.
C      DFSAMP=0.
10  CALL INPUT0
    CALL INPUT1
    CALL INPUT2
    CALL BCKGND
C      BEGIN ITERATIONS FOR VARIANCE COMPONENT DUE TO MECHANICAL CAUSES
    K=1
12  IJ=1
    VMECH=0.
    I=1

```

CONTINUED

Figure 4. Continued.

```

14 CALL LSQG
   CALL MISSLF(N)
   GO TO(20,10),N
20 CALL SANDS
   CALL SANDS1
   I=I+1
   IF(IMAX-I)25,14,14
25 CALL MECH(N)
   GO TO(40,50),N
40 K=K+1
   IF(KMAX-K)42,12,12
42 PUNCH 45
45 FORMAT(20H *CHECK CONVERGENCE)
50 GO TO(60,55),ISAMP
55 CALL SAMPL1
   CALL SAMPL2
   CALL CMPNTS
60 CALL LIMITS(STUDNT(DF))
   GO TO(10,65),ISAMP
65 CALL SPANDC
   GO TO 10
C   THE FOLLOWING ARE DUMMY STATEMENTS TO FORCE LOADING
C   OF INLINE SUBROUTINES WITH MAIN PROGRAM.
70 CALL FANDW(EXPF(SQRTF(ARSF(DF))))
   CALL EXIT
   END

```

CONTINUED

Figure 4. Continued.

```

ZZJOB
ZZDUP
*DELETFANDW
ZZFOR
*LDISK
      SUBROUTINE FANDW(N)
      W. L. WILCOXSON
      DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
1  AMDA(5),V(25),AVM(25),AVP(25)
      COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
1  A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2  AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMFCH,EMUDT,FMUO,ESAMP,I,IJ,
3  IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAXI,S2,SUMMX,TAU,TC,VEMUC,
4  VMECH,VMECHW,VSAMP
      COMMON VMECH,DFMECH,VSAMP,DFSAMP,SUMDF,ESAMP,S,EMECHS
      DIMENSION VECTOR(17)
      EQUIVALENCE(VECTOR,T)
      FETCH(IJ)VECTOR
      GO TO(70,10,10),N
10  EMUDT=0.
      DO 20 L=1,LMAX
20  EMUDT=EMUDT+EMU(I,L)*X(L)
      GO TO(70,30,70),N
30  AVPIJ=(EMUDT+ENSTAR)/T+ENSTAR/(IJ*TSTAR)
      AVMIJ=VMECHW*EMUDT**2
      W=1./((AVMIJ+(EMUDT+ENSTAR)/T+FNSTAR/TSTAR)
      IJ=IJ-1
      IF(SENSE SWITCH 3)40,60

```

CONTINUED

Figure 4. Continued.

```

40 PUNCH 50,K,IJ,AVPIJ,AVMIJ,W
50 FORMAT(2H 2,I3,I5,3E15.8)
60 RECORD(IJ)VECTOR
70 RETURN
END
ZZJOB
ZZDUP
*DELETINPUTO
ZZFOR
*LDISKINPUTO
SUBROUTINE INPUTO
C
C W. WILCOXSON
READ, TRANSFORM AND RECORD DATA
DIMENSION X(6),A(5,6),FMU(25,5),SFMU(25),VSFMU(25),JMAX(25),
1 AMDA(5),V(25),AVM(25),AVP(25)
COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
1 A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2 AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMECH,EMUDT,EMU0,ESAMP,I,IJ,
3 IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAX1,S2,SUMMX,TAU,TC,VEMU0,
4 VMECH,VMECHW,VSAMP
COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,FMECHS
EQUIVALENCE(X(1),EM),(X(2),DFM),(X(3),ES),(X(4),DFS),(X(5),EMSTRT)
READ 20
PUNCH 10
10 FORMAT(45H1 STATISTICAL ANALYSIS OF NUCLEAR DATA (SAND)/)
PUNCH 20
20 FORMAT(80H
1 READ 30,IMAX,LMAX,TAU,TC,EM,DFM,ES,DFS,EMSTRT

```

CONTINUED

Figure 4. Continued.

```

30 FORMAT(2I5,7F10.0)
   PUNCH 40,IMAX,LMAX,T0,TAU
40 FORMAT(19H  NUMBER OF SAMPLES,I14/20H  NUMBER OF ISOTOPES,I13/
   1 22H  HISTORIC TIME AT T=0,
   2 F11.2,4H MIN/19H  DETECTOR DEADTIME,F14.2,10H MIN/COUNT)
   READ 50,(AMDA(L),L=1,LMAX)
50 FORMAT(5E15.8)
   PUNCH 60,(AMDA(L),L=1,LMAX)
60 FORMAT(17H  DECAY CONSTANTS,F16.2,6H 1/MIN/(17X,F16.2))
   READ 70,(JMAX(I),I=1,IMAX)
70 FORMAT(15I5)
   PUNCH 80,(JMAX(I),I=1,IMAX)
80 FORMAT(25H  OBSERVATIONS PER SAMPLE,I8/(25X,I8))
   RETURN
   END

ZZJOB
ZZDUP
*DELETINPUT1
ZZFOR
*LDISK

      SUBROUTINE INPUT1
      W. L. WILCOXSON
      C
      C
      CONTINUATION OF INPUT ROUTINE
      DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
1  AMDA(5),V(25),AVM(25),AVP(25)
      COMMON X,W,AVMIJ,AVPIJ,ENSTAR,FN,U,TIME,CSTAR,TSTAR,C,T,
1  A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2  AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMECH,EMUDT,EMU0,ESAMP,I,IJ,

```

CONTINUED

Figure 4. Continued.

```

3  IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAX1,S2,SUMMX,TAU,T0,VEMU0, 0219
4  VMECH,VMECHW,VSAMP 0220
COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS 0221
EQUIVALENCE(X(1),EM),(X(2),DFM),(X(3),ES),(X(4),DFS),(X(5),EMSTRT) 0222
LMAX1=LMAX+1 0223
IF(FM+DFM)93,93,92 0224
92 DFMECH=DFM 0225
VMECHH=(0.01*EM)**2 0226
VMECHW=(0.01*EMSTRT)**2 0227
EMECH=EM 0228
GO TO 94 0229
93 VMECHW=(0.01*EMECH)**2 0230
VMECHH=VMECHW 0231
94 IF(VMECHW)95,95,96 0232
95 VMECHW=0.01 0233
96 VMECH=VMECHW 0234
IF(ES+DFS)110,110,100 0235
100 DFSAMP=DFS 0236
ESAMP=ES 0237
110 ESAMPS=ESAMP 0238
VSAMP=(0.01*ESAMP)**2 0239
VSAMPH=VSAMP 0240
IF(VSAMP)120,120,130 0241
120 VSAMP=0.01 0242
130 IR=1 0243
SUMDF=0. 0244
DO 140 I=1,IMAX 0245
JMAXI=JMAX(I) 0246
DF=JMAXI-LMAX 0247

```

CONTINUED

Figure 4. Continued.

```

140 SUMDF=SUMDF+DF
    PUNCH 150,EMECH,DFMECH,ESAMP,DFSAMP
150 FORMAT(27H HISTORIC MECHANICAL ERROR,F6.2,8H PERCENT/
    1 7X,18HDEGREES OF FREEDOM,I8/
    2 25H HISTORIC SAMPLING ERROR,F8.2,8H PERCENT/
    3 7X,18HDEGREES OF FREEDOM,I8)
    KMAX=25
    DELTA=0.005
    PUNCH 160,DELTA,KMAX
160 FORMAT(20H TOLERANCE ON ERROR,F13.3,8H PERCENT/
    1 28H MAXIMUM ITERATIONS ALLOWED,I5)
    RETURN
    END

ZZJOB
ZZDUP
*DELETINPUT2
ZZFOR
*LDISK

SUBROUTINE INPUT2
W. L. WILCOXSON
C
C CONTINUATION OF INPUT ROUTINE
DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
1 AMDA(5),V(25),AVM(25),AVP(25)
COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
1 A,EMU,SFEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2 AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMECH,EMUDT,FMUO,FSAMP,I,IJ,
3 IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAXI,S2,SUMMX,TAU,TC,VEMUO,

```

CONTINUED

Figure 4. Continued.

```

4 VMECH,VMECHW,VSAMP
COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
DIMENSION VECTOR(17)
EQUIVALENCE(VECTOR,T)
IJ=1
DO 60 I=1,IMAX
JMAXI=JMAX(I)
DO 60 J=1,JMAXI
READ 10,T,C,TSTAR,CSTAR,TIME
10 FORMAT(5E15.8)
TIME=TIME-TC
C TRANSFORM DEPENDENT VARIABLE AND CORRECT FOR DETECTOR DEADTIME
EN=(C/T)/(1.-TAU*(C/T))
ENSTAR=CSTAR/TSTAR
C CORRECT DEPENDENT VARIABLE FOR BACKGROUND
X(LMAXI)=EN-ENSTAR
C TRANSFORM INDEPENDENT VARIABLE AND CORRECT FOR DECAY
DO 30 L=1,LMAX
X(L)=1.
IF(AMDA(L))30,30,20
20 X(L)=EXP(-AMDA(L)*TIME)*(1.-EXP(-AMDA(L)*T))/(AMDA(L)*T)
30 CONTINUE
U=0.
IF(SENSE SWITCH 4)40,50
40 U=1.
C COMPUTE WEIGHT FOR FIRST ITERATION
50 AVPIJ=EN/T+ENSTAR/TSTAR
AVMIJ=VMECHW*X(LMAXI)**2
W=1./(AVPIJ+AVMIJ)

```

CONTINUED

Figure 4. Continued.

```

60 RECORD(IJ)VECTOR
   ISAMP=2
   IF(IMAX-1)70,70,80
70 ISAMP=1
80 RETURN
   END
ZZJOB
ZZDUP
*DELFTRCKGND
ZZFOR
*LDISK
SUBROUTINE BCKGND
W. L. WILCOXSON
COMPUTE U FOR EACH OBSERVATION
DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
1 AMDA(5),V(25),AVM(25),AVP(25)
COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIMF,CSTAR,TSTAR,C,T,
1 A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2 AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMECH,EMUDT,EMU0,ESAMP,I,IJ,
3 IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAX1,S2,SUMMX,TAU,TO,VEMU0,
4 VMECH,VMECHW,VSAMP
COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
DIMENSION VECTOR(17),VCTR(17)
EQUIVLFNCE(VECTOR,T)
EQUIVALENCE (VCTR,V)
EQUIVALENCE (VCTR(6),U1)
EQUIVALENCE (VCTR(8),ENSTR)
IF(SENSE SWITCH 4)140,1C

```

CONTINUED

Figure 4. Continued.

```

10 IJR=1
   IJM=0
   DO 130 I1=1,IMAX
     IJ0=IJM+1
     IJM=IJ0+JMAX(I1)-1
   DO 130 J1=IJ0,IJM
     FETCH(IJR)VECTOR
     IJR=IJR-1
     IJ=IJ0
   C   COMPUTE NUMBER OF TIMES BACKGROUND IJR IS USED FOR SAMPLE I1
     DO 50 K1=IJ0,IJM
       FETCH(IJ)VCTR
       IF(ENSTAR-ENSTR)50,20,50
20   IF(IJ-1-IJR)30,40,40
   C   U IS ALREADY COMPUTED FOR THIS BACKGROUND READING
30   U=U1
     GO TO 100
40   U=U+1.
50   CONTINUE
   C   COMPUTE NUMBER OF SAMPLES FOR WHICH BACKGROUND IJR IS USED
     IJ=1
     IJNXT=1
     COUNT=1.
     DO 90 I=1,IMAX
       JMAXI=JMAX(I)
       IJNXT=IJNXT+JMAXI
       IF(I-11)60,90,60

```

CONTINUED

Figure 4. Continued.

```

60 DO 70 J=1,JMAXI
   FETCH(IJ)VCTR
   IF(ENSTAR-ENSTR)70,80,70
70 CONTINUE
   GO TO 90
80 COUNT=COUNT+1.
90 IJ=IJNXT
   C   UPDATE U AND RF-RECORD RECORD
      U=U*COUNT
100 IF(SFENSE SWITCH 2)110,130
110 PUNCH 120,IJR,T,C,TSTAR,CSTAR,TIME,IJR,U,EN,ENSTAR,X(LMAX1),W,
      1 IJR,(X(L),L=1,LMAX)
120 FORMAT(2H 1,I3,5E15.8/2H 2,I3,5E15.8/2H 3,I3,5E15.8)
130 RECORD(IJR)VECTOR
140 RETURN
      END

ZZJOB
ZZDUP
*DELETLSQG
ZZFOR
*LDISK
      SUBROUTINE LSQG
      W,L,WILCOXSON
      C   GENERATE LEAST SQUARES MATRIX
      C   DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
      1 AMDA(5),V(25),AVM(25),AVP(25)
      COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,

```

CONTINUED

Figure 4. Continued.

```

1 A, EMU, SEMU, VSEMU, JMAX, AMDA, V, AVM, AVP,
2 AVMECH, AVPAM, AVPOSN, AVSAMP, DELTA, DF, EMECH, EMUDT, EMUO, ESAMP, I, IJ,
3 IMAX, IR, ISAMP, JMAXI, K, KMAX, LMAX, LMAXI, S2, SUMMX, TAU, TO, VEMUO,
4 VMECH, VMECHW, VSAMP
COMMON VMECHH, DFMFCH, VSAMPH, DFSAMP, SUMDF, ESAMPS, EMECHS
DO 10 L=1, LMAX
DO 10 LI=1, LMAXI
10 A(L, LI)=0.
JMAXI=JMAX(I)
DO 20 J=1, JMAXI
CALL FANDW(IR)
DO 20 L=1, LMAX
DO 20 LI=1, LMAXI
20 A(L, LI)=A(L, LI)+W*X(L)*X(LI)
RETURN
END

ZZJOB
ZZDUP
*DELETMISSE
ZZFOR
*LDISK

SUBROUTINE MISSE(N)
W, L, WILCOXSON
MATRIX INVERSION AND SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
DIMENSION X(6), A(5,6), EMU(25,5), SEMU(25), VSEMU(25), JMAX(25),
1 AMDA(5), V(25), AVM(25), AVP(25)
COMMON X, W, AVMIJ, AVPIJ, ENSTAR, EN, U, TIME, CSTAR, TSTAR, C, T,

MISSLE

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0412

```

CONTINUED

Figure 4. Continued.

```

1 A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2 AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,FMECH,EMUDT,EMUQ,ESAMP,I,IJ,
3 IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAX1,S2,SUMMX,TAU,TQ,VEMUQ,
4 VMECH,VMECHW,VSAMP
COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
N=1
DO 70 L=1,LMAX
TEMP=A(L,L)
A(L,L)=1.
IF(TEMP)30,10,30
10 N=2
TYPE 20
20 FORMAT(34HZERO ELEMENT ON DIAGONAL OF MATRIX/14HCASF ABANDONED)
RETURN
30 DO 40 L1=1,LMAX1
40 A(L,L1)=A(L,L1)/TEMP
DO 70 L2=1,LMAX
IF(L2-L)50,70,50
50 TEMP=A(L2,L)
A(L2,L)=0.
DO 60 L1=1,LMAX1
60 A(L2,L1)=A(L2,L1)-TEMP*A(L,L1)
70 CONTINUE
RETURN
END

```

CONTINUED

Figure 4. Continued.

```

ZZJOB
ZZDUP
*DFLETSANDS
ZZFOR
*LDISK
SUBROUTINE SANDS
C
C W. L. WILCOXSON
C
C SAVE PARAMETERS AND SUM MATRIX ELEMENTS
C
C COMPUTE TOTAL VARIANCE S2 FOR SAMPLE I
C
C PARTITION TOTAL INTO MECHANICAL AND POISSON COMPONENTS
C
C CALCULATE SEMU(I) AND ITS VARIANCE, VSEMU(I)
C
C DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
1 AMDA(5),V(25),AVM(25),AVP(25)
C
C COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
1 A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2 AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMECH,EMUDT,EMUO,ESAMP,I,IJ,
3 IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAX1,S2,SUMMX,TAU,TO,VEMUO,
4 VMECH,VMECHW,VSAMP
C
C COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
DO 10 L=1,LMAX
10 EMU(I,L)=A(L,LMAX1)
SUMSQ=0.
SUMP=0.
SUMM=0.
SUMW=0.
SUMVM=0.
IJ=IJ-JMAXI

```

CONTINUED

Figure 4. Continued.

```

DF=JMAXI-LMAX
EJMAXI=JMAXI
DO 20 J=1,JMAXI
CALL FANDW(3)
TEMP=W*(X(LMAXI)-EMUDT)**2
TEMP1=TEMP/(AVMIJ+AVPIJ)
SUMVM=SUMVM+AVMIJ*TEMP1/EMUDT**2
SUMP=SUMP+AVPIJ*TEMP1
SUMM=SUMM+AVMIJ*TEMP1
SUMSQ=SUMSQ+TEMP
20 SUMW=SUMW+W
S2=SUMSQ/(DF*SUMW)
VMECH=VMECH+EJMAXI*SUMVM/SUMW
SUMMX=0.
SEMU(I)=0.
DO 30 L=1,LMAX
SEMU(I)=SEMU(I)+EMU(I,L)
DO 30 LI=1,LMAX
A(L,LI)=SUMW*A(L,LI)
30 SUMMX=SUMMX+A(L,LI)
VSEMU(I)=S2*SUMMX
AVP(I)=SUMP*SUMMX/(DF*SUMW)
AVM(I)=SUMM*SUMMX/(DF*SUMW)
RETURN
END

```

CONTINUED

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Figure 4. Continued.

```

ZZJOB
ZZDUP
*DELETSANDS1
ZZFOR
*LDISK SANDS1
SUBROUTINE SANDS1
CONTINUATION OF SANDS SURROUTINE
W. WILCOXSON
DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
1 AMDA(5),V(25),AVM(25),AVP(25)
COMMON X,W,AVM,IJ,AVPIJ,ENSTAR,EN,U,TIMF,CSTAR,TSTAR,C,T,
1 A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2 AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,FMECH,EMUDT,FMUO,ESAMP,I,IJ,
3 IMAX,IR,ISAMP,JMAXI,K,KVAX,LMAX,LMAXI,S2,SUMMX,TAU,TO,VEMUO,
4 VMECH,VMECHW,VSAMP
COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
IF(SENSE SWITCH 1)40,90
40 DO 50 L=1,LMAX
50 PUNCH 60,I,K,(A(L,L1),L1=1,L)
60 FORMAT(2H 4,I3,I5,5E14.8)
PUNCH 70,I,K,(EMU(I,L),L=1,LMAX)
70 FORMAT(2H 5,I3,I5,5E14.8)
PUNCH 80,I,K,SEMU(I),VSEMU(I),AVP(I),AVM(I),S2
80 FORMAT(2H 6,I3,I5,5E14.8)
90 RETURN
END

```

CONTINUED

Figure 4. Continued.

```

ZZJOB
ZZDUP
*DELETMECH
ZZFOR
*LDISK
      SUBROUTINE MECH(N)
      W. L. WILCOXSON
      COMPUTE MECHANICAL COMPONENT OF ERROR
      DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
1  AMDA(5),V(25),AVM(25),AVP(25)
      COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
1  A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2  AVMECH,AVPAM,AVPOSN,AVSAMP,DFLTA,DF,FMFCH,FMUDT,FMUO,ESAMP,I,IJ,
3  IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAXI,S2,SUMMX,TAU,TC,VEMUO,
4  VMECH,VMECHW,VSAMP
      COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMP,SEMECHS
      N=1
      VMECH=VMECH/SUMDF
      EMECH0=EMECH
      EMECH=100.*SQRTF(VMECH)
      GO TO(20,40),IR
20 PUNCH 30
30 FORMAT(1H0,14X,16HMECHANICAL ERROR/
1 32H ITERATION CURRENT HISTORIC)
      IR=2
40 VMECH=(DFMECH*VMECHH+SUMDF*VMFCH)/(DFMECH+SUMDF)
      EMECHS=100.*SQRTF(VMECH)
      PUNCH 50,K,EMECH,EMECHS

```

CONTINUED

Figure 4. Continued.

```

50 FORMAT(I5,F15.2,F10.2)
IF(ABS(EMECH-EMECH0)-DELTA)90,90,100
90 N=2
100 VMECHW=VMECH
    RETURN
    END
ZZJOB
ZZDUP
*DELETSAMPL1
ZZFOR
*LDISK
SUBROUTINE SAMPL1
W. L. WILCOXSON
C
C
    CALCULATE AVERAGE DECAY RATE AT TIME TO
    DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
    1 AMDA(5),V(25),AVM(25),AVP(25)
    COMMON X,W,AVMIJ,AVPIJ,ENSTAR,FN,U,TIME,CSTAR,TSTAR,C,T,
    1 A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
    2 AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMFCH,EMUDT,FMUO,ESAMP,I,IJ,
    3 IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAXI,S2,SUMMX,TAU,TO,VEMUO,
    4 VMECH,VMECHW,VSAMP
    COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
    CALCULATE INITIAL ESTIMATE OF MEAN
    SUMV=0.
    SUM=0.
    DO 10 I=1,IMAX
    TEMPI=1./((SEMU(I)**2)*VSAMP+VSEMU(I))
    SUMV=SUMV+TEMPI

```

CONTINUED

Figure 4. Continued.

```

10 SUM=SUM+TEMP1*SEMU(I)
   EMUO=SUM/SUMV
   C   ITERATE TO FIND ESTIMATE OF MEAN BASED ON HISTORIC SAMPLING ERROR
      DO 30 K=1,KMAX
        TEMP2=FMUO
        SUMV=0.
        SUM=0.
        TEMP=VSAMP*EMUO**2
        DO 20 I=1,IMAX
          TEMP1=1./((TEMP+VSEMU(I)))
          SUMV=SUMV+TEMP1
        20 SUM=SUM+TEMP1*SEMU(I)
          FMUO=SUM/SUMV
          IF (ABS((EMUO-TEMP2)/EMUO)-0.00005)40,40,30
        30 CONTINUE
        40 PUNCH 50
        50 FORMAT(1H0,15X,14HSAMPLING ERROR/
          1 32H ITERATION CURRENT HISTORIC)
          RETURN
          END
      ZZJOB
      ZZDUP
      *DELETESAMPL2
      ZZFOR
      *LDISK
      C   SUBROUTINE SAMPL2
        W. L. WILCOXSON

```

CONTINUED

Figure 4. Continued.

```

C      BEGIN ITERATIONS FOR SAMPLING ERROR
C      AND AVERAGE COUNTS/MINUTE AT TIME TO
      DIMENSION X(6),A(5,6),EMU(25,5),SFMU(25),VSEMU(25),JMAX(25),
1     AMDA(5),V(25),AVM(25),AVP(25)
      COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
2     A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
3     AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMECH,EMUDT,EMU0,ESAMP,I,IJ,
4     IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAX1,S2,SUMMX,TAU,TO,VEMU0,
      VMECH,VMECHV,VSAMP
      COMMON VMECHH,DFMECH,VSEMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
      EI=IMAX
      DF=IMAX-1
      ESAMP=100.*SQRTF(VSAMP)
      DO 70 K=1,KMAX
        AVSAMP=VSAMP*EMU0**2
        ESAMP0=FSAMP
      COMPUTE AVERAGE DECAY RATE AT TIME TO
      SUM=0.
      SUMV=0.
      DO 10 I=1,IMAX
        TEMPI=1./((AVSAMP+VSEMU(I))
        V(I)=TEMPI
        SUMV=SUMV+TEMPI
        SUM=SUM+TEMPI*SEMU(I)
10     EMU0=SUM/SUMV
      CALCULATE ABSOLUTE SAMPLING VARIANCE
      SUMSQ=0.
      SUMSAM=0.

```

CONTINUED

Figure 4. Continued.

```

DO 20 I=1,IMAX
  TEMP1=V(I)
  V(I)=V(I)/SUMV
  TEMP2=V(I)*(SEMU(I)-EMUO)**2
  SUMSAM=SUMSAM+AVSAMP*TEMP1*TEMP2
20  SUMSQ=SUMSQ+TEMP2
  VEMUO=SUMSQ/DF
  AVSAMP=SUMSAM*EI/DF
  VSAMP=AVSAMP/EMUO**2
  ESAMP=100.*SQRTF(VSAMP)
  VSAMP=(DFSAMP*VSAMPH+DF*VSAMP)/(DFSAMP+DF)
  ESAMPS=100.*SQRTF(VSAMP)
  PUNCH 30,K,ESAMP,ESAMPS
30  FORMAT(I5,F15.2,F10.2)
  IF(SENSE SWITCH 1)40,60
40  PUNCH 50,K,EMUO,VEMUO,AVSAMP,(K,I,V(I),I=1,IMAX)
50  FORMAT(2H 8,I3,3F15.8/(2H 9,I3,I5,E15.8))
60  IF(ABS(ESAMP-ESAMPO)-DELTA)90,90,70
70  CONTINUE
  PUNCH 80
80  FORMAT(20H0 *CHECK CONVERGENCE)
90  RETURN
  END

```

CONTINUED

Figure 4. Continued.

```

ZZJOB
ZZDUP
*DELETCMPNTS
ZZFOR
*LDISK
      SUBROUTINE CMPNTS
      W. L. WILCOXSON
      C
      C
      PARTITION TOTAL VARIANCE INTO ITS THREE COMPONENTS
      DIMENSION X(6),A(5,6),EMU(25,5),SFMU(25),VSEMU(25),JMAX(25),
      1 AMDA(5),V(25),AVM(25),AVP(25)
      COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
      1 A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
      2 AVMECH,AVPAM,AVPOSN,AVSAMP,DFLTA,DF,FMECH,EMUDT,EMU0,ESAMP,I,IJ,
      3 IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAX1,S2,SUMMX,TAU,TO,VEMU0,
      4 VMECH,VMECHW,VSAMP
      COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
      SUMV2=0.
      ALPHA=0.
      BETA=0.
      DO 10 I=1,IMAX
      TEMP=V(I)**2
      SUMV2=SUMV2+TEMP
      ALPHA=ALPHA+TEMP*AVP(I)
      BETA=BETA+TEMP*AVM(I)
      GAMMA=SUMV2*VSAMP*EMU0**2
      SUM=ALPHA+BETA+GAMMA
      AVPOSN=(ALPHA/SUM)*VEMU0
10

```

CONTINUED

Figure 4. Continued.

```

AVMECH=(BETA/SUM)*VEMUC
AVSAMP=(GAMMA/SUM)*VEMUC
PUNCH 20
20 FORMAT(20H0 MEAN DECAY RATE AT TIME T=0/
1 9X,14H(ALL ISOTOPES)/6X,20HANALYSIS OF VARIANCE)
PUNCH 30,AVSAMP,AVMECH,AVPOSN,VEMUC
30 FORMAT(30H -----/
1 3CH COMPONENT VARIANCE/
2 30H -----/
3 13H SAMPLING ,F17.0/13H MECHANICAL ,F17.0/13H POISSON ,
4 F17.0/30H -----/13H TOTAL ,F17.0)
RETURN
END
ZZJOB
ZZDUP
*DELETLIMITS
ZZFOR
*LDISK
LIMITS
SUBROUTINE LIMITS(STUT)
W. L. WILCOXSON
C
C
OUTPUT MEANS AND 0.95 CONFIDENCE LIMITS ON MEANS
DIMENSION X(5),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
1 AMDA(5),V(25),AVM(25),AVP(25)
COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
1 A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
2 AVMECH,AVPAM,AVPOSN,AVSAMP,DELTA,DF,EMFCH,EMUDT,EMUC,ESAMP,I,IJ,
3 IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAXI,S2,SUMMX,TAU,TO,VEMUC,

```

CONTINUED

Figure 4. Continued.

```

4 VMECH,VMECHN,VSAMP
COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,FSAMPS,EMECHS
GO TO(2,3),ISAMP
2 VEMU0=VSEMU(1)
EMUC=SEMU(1)
3 TEMP=STUT*SQRTF(VEMU0)
TEMP1=EMU0-TEMP
TEMP2=FMUC+TEMP
PUNCH 4,TEMP1,EMU0,TEMP2
4 FORMAT(1H0,9X,35HDECAY RATE (COUNTS/MIN) AT TIME T=0/
1 9H ISOTOPE,5X,10H.95 L.C.L.,5X,9HMEAN RATE,6X,10H.95 U.C.L./
2 7H ALL,3F15.1)
IF(LMAX-1)11,11,5
C COMPUTE MEAN AND VARIANCE FOR EACH ISOTOPE
5 DO 10 L=1,LMAX
GO TO(6,7),ISAMP
6 AVG=FMU(1,L)
VAR=S2*A(L,L)
GO TO 9
7 AVG=0.
SUMSQ=0.
DO 8 I=1,IMAX
AVG=AVG+V(I)*EMU(I,L)
8 SUMSQ=SUMSQ+V(I)*EMU(I,L)**2
VAR=(SUMSQ-AVG**2)/DF
C CALCULATE CONFIDENCE LIMITS
9 TEMP=STUT*SQRTF(VAR)
TEMP1=AVG-TEMP
TEMP2=AVG+TEMP

```

CONTINUED

Figure 4. Continued.

```

10 PUNCH 12,L,TEMP1,AVG,TEMP2
12 FORMAT(4X,I2,1X,3F15.1)
11 DFMECH=DFMECH+SUMDF
   FMECH=FMECHS
   DF=IMAX-1
   DFSAMP=DFSAMP+DF
   ESAMP=ESAMPS
   PUNCH 20,EMECH,DFMECH,ESAMP,DFSAMP
20 FORMAT(35H0 UPDATED HISTORIC MECHANICAL ERROR,F6.2,8H PERCENT/
   1 15X,18HDEGREES OF FREEDOM,I8/
   2 33H UPDATED HISTORIC SAMPLING ERROR,F8.2,84 PERCENT/
   3 15X,18HDEGREES OF FREEDOM,I8)
   RETURN
   END

ZZJOB
ZZDUP
*DELETSPANDC
ZZFOR
*LDISKSPANDC
SUBROUTINE SPANDC
  W. WILCOXSON
  C
  C
  OUTPUT STATISTICS ON EACH SAMPLE
  DIMENSION X(6),A(5,6),EMU(25,5),SEMU(25),VSEMU(25),JMAX(25),
  1 AMDA(5),V(25),AVM(25),AVP(25)
  COMMON X,W,AVMIJ,AVPIJ,ENSTAR,EN,U,TIME,CSTAR,TSTAR,C,T,
  1 A,EMU,SEMU,VSEMU,JMAX,AMDA,V,AVM,AVP,
  2 AVMECH,AVPAM,AVPOSN,AVSAMP,DFLTA,DF,EMECH,FMUDT,EMU0,ESAMP,I,IJ,

```

CONTINUED

Figure 4. Continued.

```

3 IMAX,IR,ISAMP,JMAXI,K,KMAX,LMAX,LMAX1,S2,SUMMX,TAU,TO,VEMUC,
4 VMECH,VMECHW,VSAMP
COMMON VMECHH,DFMECH,VSAMPH,DFSAMP,SUMDF,ESAMPS,EMECHS
PUNCH 10,(L,L=1,5)
10 FORMAT(1H0,35X,7HISOTOPE/15X,3HALL,5I12/
18H SAMPLE,21X,40HMEAN DECAY RATE (COUNTS/MIN) AT TIME T=0)
DO 20 I=1,IMAX
20 PUNCH 30,I,SEMU(I),(EMU(I,L),L=1,LMAX)
30 FORMAT(16,2X,6F12.1)
PUNCH 40
40 FORMAT(1HC,25X,27HMEAN DECAY RATE AT TIME T=0/
125X,29H(ALL ISOTOPES - FIXED SAMPLE)/
230X,20HANALYSIS OF VARIANCE/
38H SAMPLE,3X,7HWEIGHTS,6X,10HMECHANICAL,3X,7HPOISSON,4X,5HTOTAL)
DO 50 I=1,IMAX
50 PUNCH 60,I,V(I),AVM(I),AVP(I),VSEMU(I)
60 FORMAT(16,F11.4,5X,3F11.0)
RETURN
END

ZZJOB STUDNT
ZZDUP
*DEFTSTUDNT
ZZFOR
*LDISK
FUNCTION STUDNT(DF)
C W. L. WILCOXSON
C CALCULATE STUDENTS T AT 0.95 LEVEL

```

CONTINUED

Figure 4. Continued.

```
STUDENT=12.706
IF(DF-2.12,1,3
1 STUDENT=4.303
2 RETURN
3 STUDENT=(1.96*DF+0.60033+0.95910/DF)/(DF-0.90259+0.11588/DF)
RETURN
END
ZZZZ END OF JOB
```

0787
0788
0789
0790
0791
0792
0793
0794

GLOSSARY OF SYMBOLS

A_i	Sum of all the L^2 elements of the last inverse matrix for sample i in the iterated least-squares procedure	J_i	Number of observations of the i^{th} sample.
B_{ij}	Expected value of the background for the j^{th} observation on the i^{th} sample.	j	Index to label observation; $j = 1, 2, 3, \dots, J_i$.
C, C_{ij}	Counts observed during period T, T_{ij} .	K_{ik}	A normalizing constant for the i^{th} sample on the k^{th} iteration in the computation of the weights w_{ijk} so that $\sum_{j=1}^{J_i} w_{ijk} = 1$
C^*, C_{ij}^*	Counts of background activity observed during period T^*, T_{ij}^* .	k	Iteration index used in least-squares curve-fitting procedure; also in estimating σ_s .
$\text{Cov}(a, b)$	Estimated covariance of a and b .	L	Number of radioactive isotopes in the source solution.
d	Number of degrees of freedom associated with the historic estimate of the sampling error, σ_{so} .	ℓ	Index to label isotope; $\ell = 1, 2, 3, \dots, L$.
E	The expected value or arithmetic average value operator, such that in the discrete case $E(y) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{h=1}^n y_h$, where y is any random variable and y_1, y_2, y_3, \dots are values that y takes on in successive occurrences.	M	The mean of a normally distributed random variable. Used for illustrative purposes only when discussing the mechanical error.
f_0	Number of degrees of freedom associated with the historic estimate of the mechanical error, $\tilde{\sigma}_{mo}$.	N, N_{ij}	Counts per minute from both sample and background corrected for deadtime of the detector. $N = \frac{C}{T} \left(\frac{1}{1 - \tau \frac{C}{T}} \right) \text{ and } N_{ij} = \frac{C_{ij}}{T_{ij}} \left(\frac{1}{1 - \tau \frac{C_{ij}}{T_{ij}}} \right)$
f_1	Number of degrees of freedom associated with the current estimate of the mechanical error, σ_m , based on the historic estimate σ_{mo} . $f_1 = \sum_{i=1}^I (J_i - L)$	N^*, N_{ij}^*	Apparent counts per minute from background activity. $N^* = \frac{C^*}{T^*} \text{ and } N_{ij}^* = \frac{C_{ij}^*}{T_{ij}^*}$
$f(t)$	Student's t probability density function.	P	Probability.
h	An integer; $h = 0, 1, 2, \dots$	Q	The mean of a normally distributed random variable. Used for illustrative purposes only when discussing the sampling error.
I	Number of aliquot samples of radioactive solution.	q_{ij}	A normally distributed random variable with mean zero and standard deviation σ_m labeled for the j^{th} observation of the i^{th} sample used in the simulation.
i	Index to label sample; $i = 1, 2, 3, \dots, I$.		

Number of observations of the i^{th} sample.

Index to label observation; $j = 1, 2, 3, \dots, J_i$.

A normalizing constant for the i^{th} sample on the k^{th} iteration in the computation of the weights w_{ijk} so that

$$\sum_{j=1}^{J_i} w_{ijk} = 1$$

Iteration index used in least-squares curve-fitting procedure; also in estimating σ_s

Number of radioactive isotopes in the source solution.

Index to label isotope; $l = 1, 2, 3, \dots, L$.

The mean of a normally distributed random variable. Used for illustrative purposes only when discussing the mechanical error.

Counts per minute from both sample and background corrected for deadtime of the detector.

$$N = \frac{C}{T} \left(\frac{1}{1 - \tau \frac{C}{T}} \right) \text{ and } N_{ij} = \frac{C_{ij}}{T_{ij}} \left(\frac{1}{1 - \tau \frac{C_{ij}}{T_{ij}}} \right)$$

Apparent counts per minute from background activity.

$$N^* = \frac{C^*}{T^*} \text{ and } N_{ij}^* = \frac{C_{ij}^*}{T_{ij}^*}$$

Probability.

The mean of a normally distributed random variable. Used for illustrative purposes only when discussing the sampling error.

A normally distributed random variable with mean zero and standard deviation σ_m labeled for the j^{th} observation of the i^{th} sample used in the simulation.

r_i

A normally distributed random variable with mean zero and standard deviation σ_s labeled for the i^{th} sample used in the simulation.

S_{ik}^2

For the k^{th} iteration the weighted sum of squares of deviations of the observed counts per minute for the i^{th} sample from the corresponding fitted curve divided by the degrees of freedom:

$$S_{ik}^2 = \frac{1}{J_i - L} \sum_{j=1}^{J_i} w_{ijk} [\eta_{ij} - \tilde{\mu}_{i,k}(t_{ij}, T_{ij})]^2$$

$S_{\mu,i}^2$

Estimated variance of $\tilde{\mu}_{i,i}^0$.

T, T_{ij}

Time in minutes expended in observing a sample of radioactive solution.

T^*, T_{ij}^*

Time in minutes expended in observing the background.

t, t_{ij}

Age of sample in minutes at the beginning of the observation period T or T_{ij} , where t and t_{ij} are reckoned from a historic instant of time t'_0 when $t = 0$.

t'_0

A historic instant of time when the age of all samples used in a particular experiment are reckoned to be zero minutes old; that is, $t = 0$.

t, t^*

Variable defined by Student's t density function.

U_{ij}

If a is the number of samples for which a particular background reading was used in the computations, and b is the number of times it was used just for the i^{th} sample, then $U_{ij} = ab$.

v_{ik}

Weight for the i^{th} sample on the k^{th} iteration used to determine $\tilde{\mu}_{i,i}^0$ and $\tilde{\sigma}_s$.

$\text{Var}(a)$

Estimated variance of a .

w_{ijk}

Weight for the j^{th} observation of the i^{th} sample on the k^{th} iteration.

Continued

GLOSSARY OF SYMBOLS (Continued)

$X(t, T; \lambda_{\ell})$ Independent variable in the iterated linear least-squares curve fit (corrected for decay).

$$X(t, T; \lambda_{\ell}) = e^{-\lambda_{\ell} t} \left(\frac{1 - e^{-\lambda_{\ell} T}}{\lambda_{\ell} T} \right)$$

$X_{\ell ij}$ Same as $X(t_{ij}, T_{ij}; \lambda_{\ell})$.

a_{ij} Individual point estimate of the variation due to the Poisson phenomenon for the j^{th} observation of the i^{th} sample.

$a_{..}$ Crude estimate of $\left(\frac{\tilde{\sigma}_{\mu_{..}^0}^2}{\mu_{..}^0} \right)_p$.

β_{ij} Individual point estimate of the variation due to mechanical causes for the j^{th} observation of the i^{th} sample.

$\beta_{..}$ Crude estimate of $\left(\frac{\tilde{\sigma}_{\mu_{..}^0}^2}{\mu_{..}^0} \right)_m$.

$\gamma_{..}$ Crude estimate of $\left(\frac{\tilde{\sigma}_{\mu_{..}^0}^2}{\mu_{..}^0} \right)_s$.

ϵ_{ij} Random error on the j^{th} observation of the i^{th} sample.

η, η_{ij} Assumed counts per minute arising solely from the sample. It is also the dependent variable in the regression analysis.

$$\eta = N - N^* \quad \text{and} \quad \eta_{ij} = N_{ij} - N_{ij}^*$$

κ_k A normalizing constant in the estimation of the μ_{ℓ}^0 's such that

$$\sum_{i=1}^I v_{ik} = 1$$

λ_{ℓ} A decay constant in reciprocal minutes for the ℓ^{th} isotope, $\ell = 1, 2, \dots, L$. It is assumed known and free from error in this report.

$\mu'_{\ell i}(t)$ Expected instantaneous counts per minute from isotope L of sample i at age t , corrected for background and deadtime of the detector:

$$\mu'_{\ell i}(t) = \mu_{\ell i}^0 e^{-\lambda_{\ell} t}$$

$\mu'_{\ell.}(t)$ Expected value of $\mu'_{\ell i}(t)$; that is,

$$\mu'_{\ell.}(t) = \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i=1}^I \mu'_{\ell i}(t)$$

$\mu'_{..}(t)$ Same as $\mu'_{\ell.}(t)$ except for all isotopes:

$$\mu'_{..}(t) = \sum_{\ell=1}^L \mu'_{\ell.}(t)$$

$\mu_{\ell i}(t, T)$ Expected counts per minute for a period T from isotope ℓ of sample i at age t , corrected for background and deadtime of the detector.

$\mu_{.i}(t, T)$ Same as $\mu_{\ell i}(t, T)$ except for all isotopes in the sample; that is,

$$\mu_{.i}(t, T) = \sum_{\ell=1}^L \mu_{\ell i}(t, T)$$

$\mu_{\ell.}(t, T)$ The population mean of $\mu_{\ell i}(t, T)$; that is,

$$\mu_{\ell.}(t, T) = \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i=1}^I \mu_{\ell i}(t, T)$$

$\mu_{..}(t, T)$ The population mean of $\mu_{.i}(t, T)$; that is,

$$\mu_{..}(t, T) = \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i=1}^I \mu_{.i}(t, T)$$

$$= \sum_{\ell=1}^L \mu_{\ell.}(t, T)$$

μ_{ℓ}^0

$\mu_{.i}^0$

$\mu_{\ell.}^0$

$\mu_{..}^0$

ξ_{ij}

σ^2

σ_m

$\mu'_{\ell i}(t)$ Expected instantaneous counts per minute from isotope ℓ of sample i at age t , corrected for background and deadtime of the detector:

$$\mu'_{\ell i}(t) = \mu_{\ell i}^0 e^{-\lambda_{\ell} t}$$

$\mu'_{\ell.}(t)$ Expected value of $\mu'_{\ell i}(t)$; that is,

$$\mu'_{\ell.}(t) = \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i=1}^I \mu'_{\ell i}(t)$$

$\mu'_{..}(t)$ Same as $\mu'_{\ell.}(t)$ except for all isotopes:

$$\mu'_{..}(t) = \sum_{\ell=1}^L \mu'_{\ell.}(t)$$

$\mu_{\ell i}(t, T)$ Expected counts per minute for a period T from isotope ℓ of sample i at age t , corrected for background and deadtime of the detector.

$\mu_{.i}(t, T)$ Same as $\mu_{\ell i}(t, T)$ except for all isotopes in the sample; that is,

$$\mu_{.i}(t, T) = \sum_{\ell=1}^L \mu_{\ell i}(t, T)$$

$\mu_{\ell.}(t, T)$ The population mean of $\mu_{\ell i}(t, T)$; that is,

$$\mu_{\ell.}(t, T) = \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i=1}^I \mu_{\ell i}(t, T)$$

$\mu_{..}(t, T)$ The population mean of $\mu_{.i}(t, T)$; that is,

$$\begin{aligned} \mu_{..}(t, T) &= \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i=1}^I \mu_{.i}(t, T) \\ &= \sum_{\ell=1}^L \mu_{\ell.}(t, T) \end{aligned}$$

$\mu_{\ell i}^0$

Crudely the expected counts per minute at time $t = 0$ from isotope ℓ of sample i :

$$\mu_{\ell i}^0 = \lim_{T \rightarrow \infty} \mu_{\ell i}(0, T)$$

$\mu_{.i}^0$

Same as $\mu_{\ell i}^0$ except for all isotopes of sample i :

$$\mu_{.i}^0 = \sum_{\ell=1}^L \mu_{\ell i}^0$$

$\mu_{\ell.}^0$

The population mean of $\mu_{\ell i}^0$; that is,

$$\mu_{\ell.}^0 = \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i=1}^I \mu_{\ell i}^0$$

$\mu_{..}^0$

Same as $\mu_{\ell.}^0$ except for all isotopes:

$$\mu_{..}^0 = \sum_{\ell=1}^L \mu_{\ell.}^0$$

ξ_{ij}

For the j th observation of the i th sample of the generated data, an intermediate quantity (counts per minute) that includes the effects of both sampling and mechanical errors but is free of Poisson effects.

$$\xi_{ij} = (1 + r_i)(1 + a_{ij})\mu_{.i}(t_{ij}, T_{ij})$$

The Poisson parameter $T_{ij}(\xi_{ij} + B_{ij})$ is required to generate C_{ij} .

σ^2

Used in general to indicate a variance (may be proportional in some cases). For example σ_a^2 is used to designate the variance of the quantity a . Likewise, $\tilde{\sigma}_a^2$ would be an estimate of the variance of a .

σ_m

Proportional mechanical error; that is, a proportional error due to all causes other than sampling error and errors that result from the Poisson phenomenon.

Continued

B.

GLOSSARY OF SYMBOLS (Continued)

$\tilde{\sigma}_{mk}$	An estimate of σ_m on the k^{th} iteration of the least-squares process. $\tilde{\sigma}_{m0}$ is a historical estimate required for starting the iterated least-squares process.
$\tilde{\sigma}_{m0}$	The appropriate historic estimate of the proportional mechanical error based on f_0 degrees of freedom.
$\sigma_{m'}$	The current proportional mechanical error based on $\tilde{\sigma}_{m0}$.
$\tilde{\sigma}_{m'k}$	An estimate of $\sigma_{m'}$ on the k^{th} iteration.
σ_s	Proportional sampling error.
$\tilde{\sigma}_{sk}$	An estimate of σ_s on the k^{th} iteration of the least-squares process.
$\tilde{\sigma}_{s0}$	The appropriate historic estimate of the proportional sampling error based on d degrees of freedom.
$\sigma_{s'}$	The current proportional sampling error based on $\tilde{\sigma}_{s0}$.
$\tilde{\sigma}_{s'k}$	An estimate of $\sigma_{s'}$ on the k^{th} iteration.
$\tilde{\sigma}_{\mu_{..}^0}^2$	Total estimated variance of $\tilde{\mu}_{..}^0$.
	$\tilde{\sigma}_{\mu_{..}^0}^2 = \left(\tilde{\sigma}_{\mu_{..}^0}^2 \right)_m + \left(\tilde{\sigma}_{\mu_{..}^0}^2 \right)_p + \left(\tilde{\sigma}_{\mu_{..}^0}^2 \right)_s$
$\left(\tilde{\sigma}_{\mu_{..}^0}^2 \right)_m$	Estimated variance of $\tilde{\mu}_{..}^0$ due to mechanical effects.
$\left(\tilde{\sigma}_{\mu_{..}^0}^2 \right)_p$	Estimated variance of $\tilde{\mu}_{..}^0$ due to Poisson causes.
$\left(\tilde{\sigma}_{\mu_{..}^0}^2 \right)_s$	Estimated variance of $\tilde{\mu}_{..}^0$ due to sampling effects.
τ	Deadtime of the detector in minutes.

o

When used as a superscript quantity essentially at the

$$\mu_{21}^0 = E \left[\lim_{T \rightarrow 0} \frac{1}{T} \int_0^T \mu_{21} \right]$$

When used as a subscript for that particular subscript usage is usually different

A bar placed over any symbol arithmetic average or the number of items; for example

$$\bar{v} = \frac{1}{n} \sum_{n=1}^n v_n$$

~

A tilde placed over any symbol an estimate of the true value $\tilde{\mu}_{21}^0$ is an estimate of μ_{21}^0

Y OF SYMBOLS (Continued)

An estimate of σ_m on the k^{th} iteration of the least-squares process. $\tilde{\sigma}_{m0}$ is a historical estimate required for starting the iterated least-squares process.

The appropriate historic estimate of the proportional mechanical error based on f_0 degrees of freedom.

The current proportional mechanical error based on $\tilde{\sigma}_{m0}$.

An estimate of σ_m on the k^{th} iteration.

Proportional sampling error.

An estimate of σ_s on the k^{th} iteration of the least-squares process.

The appropriate historic estimate of the proportional sampling error based on d degrees of freedom.

The current proportional sampling error based on $\tilde{\sigma}_{s0}$.

An estimate of σ_s on the k^{th} iteration.

Total estimated variance of $\tilde{\mu}_{..}^0$.

$$\tilde{\sigma}_{\tilde{\mu}_{..}^0}^2 = \left(\tilde{\sigma}_{\tilde{\mu}_{..}^0}^2 \right)_m + \left(\tilde{\sigma}_{\tilde{\mu}_{..}^0}^2 \right)_p + \left(\tilde{\sigma}_{\tilde{\mu}_{..}^0}^2 \right)_s$$

Estimated variance of $\tilde{\mu}_{..}^0$ due to mechanical effects.

Estimated variance of $\tilde{\mu}_{..}^0$ due to Poisson causes.

Estimated variance of $\tilde{\mu}_{..}^0$ due to sampling effects.

Deadtime of the detector in minutes.

o

When used as a superscript, it indicates the quantity essentially at time $t = 0$; for example,

$$\mu_{Q1}^0 = E \left[\lim_{T \rightarrow 0} \frac{1}{T} \int_0^T \mu_{Q1}(t, T) dt \right]$$

.

When used as a subscript, it acts as a placeholder for that particular subscript. Interpretation of usage is usually different in each case.

—

A bar placed over any symbol indicates an arithmetic average or the mean of a finite number of items; for example,

$$\bar{y} = \frac{1}{n} \sum_{h=1}^n y_h$$

~

A tilde placed over any quantity indicates it is an estimate of the true quantity; for example,

$\tilde{\mu}_{Q1}^0$ is an estimate of μ_{Q1}^0 .

<p>Naval Civil Engineering Laboratory ESTIMATING STRENGTHS OF INDIVIDUAL RADIOISOTOPES IN A MULTIPLE-ISOTOPE SOURCE, by M. L. Eaton and W. L. Wilcoxson TR-551 81 p. illus Nov 1967 Unclassified</p> <p>1. Statistical analysis 2. Radioisotope decay I. Z-R011-01-01-101</p> <p>In work related to radiation shielding, the use of radioisotope techniques, and activation analysis, an experimenter must often analyze counting data where counts are caused by the natural background and by the decay of more than one radioisotope. In this report a procedure is developed for estimating the strength of each isotope at different times from several decaying radioactive samples of a single multiple-isotope source. In addition, the procedure provides a method for placing confidence limits on the strengths and a method for partitioning the imprecision of estimating the strengths into three principal causes: Poisson variation, sampling error, and residual error (called mechanical error).</p> <p>An operational FORTRAN II-D computer program, SAND, implements the procedure. The procedure and program were tested by using fictitious data with known properties as inputs. The results of the simulation were in reasonable agreement with the theoretical values.</p>	<p>Naval Civil Engineering Laboratory ESTIMATING STRENGTHS OF INDIVIDUAL RADIOISOTOPES IN A MULTIPLE-ISOTOPE SOURCE, by M. L. Eaton and W. L. Wilcoxson TR-551 81 p. illus Nov 1967 Unclassified</p> <p>1. Statistical analysis 2. Radioisotope decay I. Z-R011-01-01-101</p> <p>In work related to radiation shielding, the use of radioisotope techniques, and activation analysis, an experimenter must often analyze counting data where counts are caused by the natural background and by the decay of more than one radioisotope. In this report a procedure is developed for estimating the strength of each isotope at different times from several decaying radioactive samples of a single multiple-isotope source. In addition, the procedure provides a method for placing confidence limits on the strengths and a method for partitioning the imprecision of estimating the strengths into three principal causes: Poisson variation, sampling error, and residual error (called mechanical error).</p> <p>An operational FORTRAN II-D computer program, SAND, implements the procedure. The procedure and program were tested by using fictitious data with known properties as inputs. The results of the simulation were in reasonable agreement with the theoretical values.</p>
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Security Classification

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Radioisotopes						
Radiation shielding						
Statistical analysis						
Counting data						
Radioactive sample decay						
Multiple-isotope source						
Poisson variation						
Sampling error						
Residual error						
Mechanical error						
FORTAN II-D computer						
Nuclear physics						

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